~~Miss Annie M. French~~

with the
compliments of the
author.

THE

FIRST STEPS IN ALGEBRA

BY

G. A. WENTWORTH, A.M.

AUTHOR OF A SERIES OF TEXT-BOOKS IN MATHEMATICS

TEACHERS' EDITION

BOSTON, U.S.A.

PUBLISHED BY GINN & COMPANY

1894

Miss Ellen L. Wentworth
Exeter, N. H.

The Athenæum Press
Ginn & Company

PREFACE.

THIS edition is intended for teachers, and for them only. The publishers will make every effort to keep the book from pupils; and teachers are urged to exercise the utmost care not to lose their copies, or to leave them where pupils can have access to them.

It is hoped that young teachers will derive great advantage from studying the systematic arrangement of the algebraic work, for such attention has been paid to this as the limitation of the page would allow. It is of the utmost importance to teach pupils the neatest and most orderly arrangement of work, and to require them to give at the left on each line a statement of the step taken. In this way most of the seeming difficulties of problems vanish.

It is also expected that many teachers will find great relief by not being obliged to work out every problem in the Algebra.

G. A. WENTWORTH.

EXETER, N. H.,
June, 1894.

FIRST STEPS IN ALGEBRA.

TEACHERS' EDITION.

Exercise 1. Page 10.

Remove the parentheses, and combine:

1. $9 + (3 + 2)$.
   $9 + (3 + 2) = 9 + 3 + 2 = 14$.
2. $9 + (3 - 2)$.
   $9 + (3 - 2) = 9 + 3 - 2 = 10$.
3. $7 + (5 + 1)$.
   $7 + (5 + 1) = 7 + 5 + 1 = 13$.
4. $7 + (5 - 1)$.
   $7 + (5 - 1) = 7 + 5 - 1 = 11$.
5. $6 + (4 + 3)$.
   $6 + (4 + 3) = 6 + 4 + 3 = 13$.
6. $6 + (4 - 3)$.
   $6 + (4 - 3) = 6 + 4 - 3 = 7$.
7. $3 + (8 - 2)$.
   $3 + (8 - 2) = 3 + 8 - 2 = 9$.
8. $9 - (8 - 6)$.
   $9 - (8 - 6) = 9 - 8 + 6 = 7$.
9. $10 - (9 - 5)$.
   $10 - (9 - 5) = 10 - 9 + 5 = 6$.
10. $9 - (6 + 1)$.
    $9 - (6 + 1) = 9 - 6 - 1 = 2$.
11. $8 - (3 + 2)$.
    $8 - (3 + 2) = 8 - 3 - 2 = 3$.

12. $7 - (3 - 2)$.
$7 - (3 - 2) = 7 - 3 + 2 = 6$.

13. $9 - (4 + 3)$.
$9 - (4 + 3) = 9 - 4 - 3 = 2$.

14. $9 - (4 - 3)$.
$9 - (4 - 3) = 9 - 4 + 3 = 8$.

15. $7 - (5 - 2)$.
$7 - (5 - 2) = 7 - 5 + 2 = 4$.

16. $7 - (7 - 3)$.
$7 - (7 - 3) = 7 - 7 + 3 = 3$.

17. $(8 - 6) - 1$.
$(8 - 6) - 1 = 8 - 6 - 1 = 1$.

18. $(3 - 2) - (1 - 1)$.
$(3 - 2) - (1 - 1) = 3 - 2 - 1 + 1 = 1$.

19. $(7 - 3) - (3 - 2)$.
$(7 - 3) - (3 - 2) = 7 - 3 - 3 + 2 = 3$.

20. $(8 - 2) - (5 - 3)$.
$(8 - 2) - (5 - 3) = 8 - 2 - 5 + 3 = 4$.

21. $15 - (10 - 3 - 2)$.
$15 - (10 - 3 - 2) = 15 - 10 + 3 + 2 = 10$.

NOTE. — The teacher should make sure that every pupil is able to remove parentheses without making mistakes in *signs*, before he leaves this subject. If the examples are not sufficient for this purpose, he must put more on the board, until practice makes every pupil perfect in this matter.

Exercise 2. Page 12.

Multiply:

1. $7(8 + 5)$.
$7(8 + 5) = 7 \times 13 = 91$.
$7(8 + 5) = 56 + 35 = 91$.

2. $7(8 - 5)$.
$7(8 - 5) = 7 \times 3 = 21$.
$7(8 - 5) = 56 - 35 = 21$.

3. $6(7 + 3)$.
$6(7 + 3) = 6 \times 10 = 60$.
$6(7 + 3) = 42 + 18 = 60$.

4. $6(7 - 3)$.
$6(7 - 3) = 6 \times 4 = 24$.
$6(7 - 3) = 42 - 18 = 24$.

5. $8(7+5)$.
$8(7+5) = 8 \times 12 = 96$.
$8(7+5) = 56 + 40 = 96$.

6. $8(7-5)$.
$8(7-5) = 8 \times 2 = 16$.
$8(7-5) = 56 - 40 = 16$.

7. $9(6-2)$.
$9(6-2) = 9 \times 4 = 36$.
$9(6-2) = 54 - 18 = 36$.

8. $4(a+b)$.
$4(a+b) = 4a + 4b$.

9. $4(a-b)$.
$4(a-b) = 4a - 4b$.

10. $2(a^2+b^2)$.
$2(a^2+b^2) = 2a^2 + 2b^2$.

11. $2(a^2-b^2)$.
$2(a^2-b^2) = 2a^2 - 2b^2$.

12. $3(ab+c)$.
$3(ab+c) = 3ab + 3c$.

13. $3(ab-c)$.
$3(ab-c) = 3ab - 3c$.

14. $3(c-ab)$.
$3(c-ab) = 3c - 3ab$.

15. $a(b+c)$.
$a(b+c) = ab + ac$.

16. $a(b-c)$.
$a(b-c) = ab - ac$.

17. $3a(b+c)$.
$3a(b+c) = 3ab + 3ac$.

18. $3a(b-c)$.
$3a(b-c) = 3ab - 3ac$.

19. $5a(b^2+c)$.
$5a(b^2+c) = 5ab^2 + 5ac$.

20. $5a(b^2-c^2)$.
$5a(b^2-c^2) = 5ab^2 - 5ac^2$.

21. $5a^2(b^2-c)$.
$5a^2(b^2-c) = 5a^2b^2 - 5a^2c$.

NOTE. — Problems from 1 to 7 illustrate the *Distributive Law* in multiplication; that is, they illustrate the fact that multiplying the separate terms of the multiplicand by the multiplier, and combining the partial products, produces the same result as combining the terms of the multiplicand first (when this is possible) and then multiplying by the multiplier.

Exercise 3. Page 12.

If $a = 7$, $b = 5$, $c = 3$, find the value of:

1. $9a$.
$9a = 9 \times 7 = 63$.

2. $8ab$.
$8ab = 8 \times 7 \times 5 = 280$.

3. $4b^2c$.
$4b^2c = 4 \times 5 \times 5 \times 3 = 300$.

4. $2a^2$.
$2a^2 = 2 \times 7 \times 7 = 98$.

5. $3c^3$.
$3c^3 = 3 \times 3 \times 3 \times 3 = 81$.

6. $2b^4$.
$2b^4 = 2 \times 5 \times 5 \times 5 \times 5 = 1250$.

7. $5ac$.
$5ac = 5 \times 7 \times 3 = 105$.

8. abc.
$abc = 7 \times 5 \times 3 = 105$.

9. abc^2.
$abc^2 = 7 \times 5 \times 3 \times 3 = 315$.

10. $\frac{1}{3}abc$.
$\frac{1}{3}abc = \frac{1}{3} \times 7 \times 5 \times 3 = 35$.

11. $\frac{1}{5}ab^2c$.
$\frac{1}{5}ab^2c = \frac{1}{5} \times 7 \times 5 \times 5 \times 3 = 105$.

12. $\frac{1}{7}a^2bc$.
$\frac{1}{7}a^2bc = \frac{1}{7} \times 7 \times 7 \times 5 \times 3 = 105$.

If $a = 5$, $b = 2$, $c = 0$, $x = 1$, $y = 3$, find the value of:

13. $4acy^2$.
$4acy^2 = 4 \times 5 \times 0 \times (3)^2 = 0$.

14. $3ax^5y^2$.
$3ax^5y^2 = 3 \times 5 \times (1)^5 \times (3)^2 = 135$.

15. $2ab^2y$.
$2ab^2y = 2 \times 5 \times (2)^2 \times 3 = 120$.

16. $2a^2b^2c^2y^2$.
$2a^2b^2c^2y^2 = 2 \times (5)^2 \times (2)^2 \times (0)^2 \times (3)^2 = 0$.

17. $2a^2b^2x^2y^2$.
$2a^2b^2x^2y^2 = 2 \times (5)^2 \times (2)^2 \times (1)^2 \times (3)^2 = 1800$.

18. $2abx^3y^3$.
$2abx^3y^3 = 2 \times 5 \times 2 \times (1)^3 \times (3)^3 = 540$.

19. $3abcxy$.
$3abcxy = 3 \times 5 \times 2 \times 0 \times 1 \times 3 = 0$.

20. $3abx^3y^2$.
$3abx^3y^2 = 3 \times 5 \times 2 \times (1)^3 \times (3)^2 = 270$.

21. $3ab^2xy^2$.
$3ab^2xy^2 = 3 \times 5 \times (2)^2 \times 1 \times (3)^2 = 540$.

Exercise 4. Page 13.

If $a = 5$, $b = 4$, $c = 3$, find the value of:

1. $9a - 2bc$.
$9a - 2bc = 45 - 24 = 21$.

2. $ab + 2c$.
$ab + 2c = 20 + 6 = 26$.

3. $abc + bc$.
$abc + bc = 60 + 12 = 72$.

4. $5ac + 2a$.
$5ac + 2a = 75 + 10 = 85$.

5. $2abc - 2ac^2$.
$2abc - 2ac^2 = 120 - 90 = 30$.

6. $ab + bc - ac$.
$ab + bc - ac = 20 + 12 - 15 = 17$.

7. $ac - (b + c)$.
$ac - (b + c) = 15 - 7 = 8$.

8. $a^2 + (b^2 + c^2)$.
$a^2 + (b^2 + c^2) = 25 + 25 = 50$.

9. $2a + (2b + 2c)$.
$2a + (2b + 2c) = 10 + 14 = 24$.

10. $a^2 - b^2 - c^2$.
$a^2 - b^2 - c^2 = 25 - 16 - 9 = 0$.

11. $3(a - b + c)$.
$3(a - b + c) = 3 \times 4 = 12$.

12. $6ab - (bc + 8)$.
$6ab - (bc + 8) = 120 - 20 = 100$.

13. $7bc - c^2 + a$.
$7bc - c^2 + a = 84 - 9 + 5 = 80$.

14. $5ac - b^2 + 3b$.
$5ac - b^2 + 3b = 75 - 16 + 12 = 71$.

15. $4b^2c - 5c^2 - 2b$.
$4b^2c - 5c^2 - 2b = 192 - 45 - 8 = 139$.

16. $2a + (b + c)$.
$2a + (b + c) = 10 + 7 = 17$.

17. $b + 2(a - c)$.
$b + 2(a - c) = 4 + 4 = 8$.

18. $c + 2(a - b)$.
$c + 2(a - b) = 3 + 2 = 5$.

19. $2a - (b + c)$.
$2a - (b + c) = 10 - 7 = 3$.

20. $2b - (a - c)$.
$2b - (a - c) = 8 - 2 = 6$.

21. $2c - (a - b)$.
$2c - (a - b) = 6 - 1 = 5$.

22. $2c - 5(a - b)$.
$2c - 5(a - b) = 6 - 5 = 1$.

23. $2b - 3(a - c)$.
$2b - 3(a - c) = 8 - 6 = 2$.

24. $2c - b(a - b)$.
$2c - b(a - b) = 6 - 4 = 2$.

Exercise 5. Page 14.

1. Read $a+b$; $a-b$; ab; $a \div b$.
a plus b; a minus b; a times b; a divided by b.

2. Write six increased by four. $6+4$.

3. Write a increased by b. $a+b$.

4. Write six diminished by four. $6-4$.

5. Write a diminished by b. $a-b$.

6. By how much does twenty-five exceed sixteen? $25-16$.

7. By how much does x exceed y? $x-y$.

8. Write four times three; the fourth power of three. 4×3; 3^4.

9. Write four times x; the fourth power of x. $4x$; x^4.

10. If one *part* of twenty-five is fifteen, what is the other part? $25-15$.

11. If one part of 35 is x, what is the other part? $35-x$.

12. If one part of x is a, what is the other part? $x-a$.

13. How much does ten lack of being twelve? $12-10$.

14. How much does x lack of being fourteen? $14-x$.

15. How much does x lack of being a? $a-x$.

16. If a man walks four miles an hour, how many miles will he walk in three hours? 3×4.

17. If a man walks y miles an hour, how many miles will he walk in x hours? xy.

18. If a man walks y miles an hour, how many hours will it take him to walk x miles? $\frac{x}{y}$.

Exercise 6. Page 15.

1. If the dividend is twenty and the quotient five, what is the divisor? $\frac{20}{5}$.

2. If the dividend is a and the quotient b, what is the divisor? $\frac{a}{b}$.

3. If John is twenty years old to-day, how old was he four years ago? How old will he be five years hence? $(20-4)$ yr.; $(20+5)$ yr.

4. If James is x years old to-day, how old was he three years ago? How old will he be seven years hence? $(x-3)$ yr.; $(x+7)$ yr.

5. Write four times the *expression* seven minus five. $4(7-5)$.

6. Write seven times the *expression* $2x$ minus y. $7(2x-y)$.

7. Write the next integral number above four. $4+1$.

8. If x is an integral number, write the next integral number above it; the next integral number below it. $x+1$; $x-1$.

9. What number is less than 20 by d? $20-d$.

10. If the difference of two numbers is five, and the smaller number is fifteen, what is the greater number? $15+5$.

11. If the difference of two numbers is eight, and the smaller number is x, what is the greater number? $x+8$.

12. If the sum of two numbers is 30, and one of them is 20, what is the other? $30-20$.

13. If the sum of two numbers is x, and one of them is 10, what is the other? $x-10$.

14. If 100 contains x ten times, what is the value of x? 10.

Exercise 7. Page 16.

1. In x years a man will be 40 years old; what is his present age? $(40-x)$ years.

2. How old will a man be in y years, if his present age is a years? $(a+y)$ years.

3. What is the value of x if $7x$ equals 28? $\frac{28}{7}$, or 4.

4. If it takes 3 men 4 days to reap a field, how many days will it take one man to reap it? 3×4 days.

5. If it takes a men b days to reap a field, how many days will it take one man to do it? ab days.

6. What is the excess of $5x$ over $3x$? $5x-3x$, or $2x$.

7. By how much does $20-3$ exceed $(10+1)$? $20-3-(10+1)$, or 6.

8. By how much does $2x-3$ exceed $(x+1)$? $(2x-3)-(x+1)$, or $x-4$.

9. If x stands for 10, find the value of $4(3x-20)$. 4×10, or 40.

10. If a stands for 10, and b for 2, find the value of $2(a-2b)$. $2(10-4)$, or 12.

11. How many cents in a dollars, b quarters, and c dimes? $100a + 25b + 10c$.

12. A book-shelf contains French, Latin, and Greek books. There are 100 books in all, and there are x Latin and y Greek books. How many French books are there? $100 - x - y$.

13. A regiment of men is drawn up in 10 ranks of 80 men each, and there are 15 men over. How many men are there in the regiment? $10 \times 80 + 15$.

14. A regiment of men is drawn up in x ranks of y men each, and there are c men over. How many men are there in the regiment? $xy + c$.

Exercise 8. Page 17.

1. A room is 10 yards long and 8 yards wide. In the middle there is a carpet 6 yards square. How many square yards of oilcloth will be required to cover the rest of the floor? $10 \times 8 - 6^2$.

2. A room is x yards long and y yards wide. In the middle there is a carpet a yards square. How many square yards of oilcloth will be required to cover the rest of the floor? $xy - a^2$.

3. How many rolls of paper g feet long and k feet wide will be required to paper a room, the perimeter of which, after proper allowance is made for doors and windows, is p feet and the height h feet? $\frac{ph}{kg}$.

4. Write six times the square of m, plus five c times the expression d plus b minus a. $6m^2 + 5c(d + b - a)$.

5. Write five times the expression two n plus one, diminished by six times the expression c minus a plus b. $5(2n + 1) - 6(c - a + b)$.

6. A lady bought a dress for a dollars, a cloak for b dollars, two pairs of gloves for c dollars a pair. She gave a hundred-dollar bill in payment. How much money should be returned to her? $\$100 - \$(a + b + 2c)$.

7. If a man can perform a piece of work in 4 days, how much of it can he do in one day? $\frac{1}{4}$.

8. If a man can perform a piece of work in x days, how much of it can he do in one day? $\frac{1}{x}$.

9. If A can do a piece of work in x days, B in y days, C in z days, how much of it can they all do in one day, working together? $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}.$

10. Write an expression for the sum, and also for the product, of three consecutive numbers of which the least is n.

$$\text{Sum} = n + (n+1) + (n+2).$$
$$\text{Product} = n(n+1)(n+2).$$

11. The product of two factors is 36; if one of the factors is x, what is the other factor? $\frac{36}{x}.$

12. If d is the divisor and q the quotient, what is the dividend? $qd.$

13. If d is the divisor, q the quotient, and r the remainder, what is the dividend? $qd + r.$

14. If x oranges can be bought for 50 cents, how many oranges can be bought for 100 cents? $2x.$

15. What is the price in cents of x apples, if they are ten cents a dozen? $\frac{10x}{12}.$

16. If b oranges cost 6 cents, what will a oranges cost? $\frac{6a}{b}.$

17. How many miles between two places, if a train travelling m miles an hour requires 4 hours to make the journey? $4m.$

18. If a man was x years old 10 years ago, how many years old will he be 7 years hence? $x + 10 + 7.$

19. If a man was x years old y years ago, how many years old will he be c years hence? $x + y + c.$

20. If a floor is $3x$ yards long and 12 yards wide, how many square yards does the floor contain? $36x.$

21. How many hours will it take to walk c miles, at the rate of one mile in 15 minutes? $\frac{c}{4}.$

22. Write three consecutive numbers of which x is the middle number. $x - 1, x, x + 1.$

23. If an odd number is represented by $2n + 1$, what will represent the next odd number? $2n + 3.$

Exercise 9. Page 24.

Find the number that x stands for if :

1. $3x = x + 8.$

$$3x = x + 8$$
$$2x = 8$$
$$\therefore x = 4.$$

2. $3x = 2x + 5.$

$$3x = 2x + 5$$
$$\therefore x = 5.$$

3. $3x + 4 = x + 10.$

$$3x + 4 = x + 10$$
$$3x - 4 = 10 - 4$$
$$2x = 6$$
$$\therefore x = 3.$$

4. $4x + 6 = x + 9.$

$$4x + 6 = x + 9$$
$$4x - x = 9 - 6$$
$$3x = 3$$
$$\therefore x = 1.$$

5. $7x - 19 = 5x + 7.$

$$7x - 19 = 5x + 7$$
$$7x - 5 = 7 + 19$$
$$2x = 26$$
$$\therefore x = 13.$$

6. $3(x - 2) = 2(x - 3).$

$$3(x - 2) = 2(x - 3)$$
$$3x - 6 = 2x - 6$$
$$3x - 2x = 6 - 6$$
$$\therefore x = 0.$$

7. $8x + 7 = 4x + 27.$

$$8x + 7 = 4x + 27$$
$$8x - 4x = 27 - 7$$
$$4x = 20$$
$$\therefore x = 5.$$

8. $3x + 10 = x + 20.$

$$3x + 10 = x + 20$$
$$3x - x = 20 - 10$$
$$2x = 10$$
$$\therefore x = 5.$$

9. $5(x - 2) = 3x + 4.$

$$5(x - 2) = 3x + 4$$
$$5x - 10 = 3x + 4$$
$$5x - 3x = 4 + 10$$
$$2x = 14$$
$$\therefore x = 7.$$

10. $3(x - 2) = 2(x - 1).$

$$3(x - 2) = 2(x - 1)$$
$$3x - 6 = 2x - 2$$
$$3x - 2x = 6 - 2$$
$$\therefore x = 4.$$

11. $2x + 3 = 16 - (2x - 3).$

$$2x + 3 = 16 - (2x - 3)$$
$$2x + 3 = 16 - 2x + 3$$
$$2x + 2x = 16 + 3 - 3$$
$$4x = 16.$$
$$\therefore x = 4.$$

12. $19x - 3 = 2(7 + x).$

$$19x - 3 = 2(7 + x)$$
$$19x - 3 = 14 + 2x$$
$$19x - 2x = 14 + 3$$
$$17x = 17$$
$$\therefore x = 1.$$

13. $7x - 70 = 5x - 20.$

$$7x - 70 = 5x - 20$$
$$7x - 5x = 70 - 20$$
$$2x = 50$$
$$\therefore x = 25.$$

14. $2x - 22 = 108 - 2x.$

$$2x - 22 = 108 - 2x$$
$$2x + 2x = 108 + 22$$
$$4x = 130$$
$$x = 32\tfrac{1}{2}.$$

15. $2(x+5) + 5(x-4) = 32.$

$$2(x+5) + 5(x-4) = 32$$
$$2x + 10 + 5x - 20 = 32$$
$$2x + 5x = 32 - 10 + 20$$
$$7x = 42$$
$$\therefore x = 6.$$

16. $2(3x - 25) = 10.$

$$2(3x - 25) = 10$$
$$6x - 50 = 10$$
$$6x = 60$$
$$\therefore = 10.$$

17. $33x - 70 = 3x + 20.$

$$33x - 70 = 3x + 20$$
$$33x - 3x = 20 + 70$$
$$30x = 90$$
$$\therefore x = 3.$$

18. $4(1+x) + 3(2+x) = 17.$

$$4(1+x) + 3(2+x) = 17$$
$$4 + 4x + 6 + 3x = 17$$
$$4x + 3x = 17 - 4 - 6$$
$$7x = 7$$
$$\therefore x = 1.$$

19. $8x - (x+2) = 47.$

$$8x - (x+2) = 47$$
$$8x - x - 2 = 47$$
$$7x = 49$$
$$\therefore x = 7.$$

20. $3(x-2) = 50 - (2x - 9).$

$$3(x-2) = 50 - (2x - 9)$$
$$3x - 6 = 50 - 2x + 9$$
$$3x + 2x = 50 + 9 + 6$$
$$5x = 65$$
$$\therefore x = 13.$$

21. $2x - (3 + 4x - 3x + 5) = 4.$

$$2x - (3 + 4x - 3x + 5) = 4$$
$$2x - 3 - 4x + 3x - 5 = 4$$
$$2x - 4x + 3x = 4 + 3 + 5$$
$$\therefore x = 12.$$

22. $5(2-x) + 7x - 21 = x + 3.$

$$5(2-x) + 7x - 21 = x + 3$$
$$10 - 5x + 7x - 21 = x + 3$$
$$-5x + 7x - x = 3 - 10 + 21$$
$$\therefore x = 14.$$

23. $(3x-2) + 2(x-3) + (x-4) = 3x + 5.$

$$3(x-2) + 2(x-3) + (x-4) = 3x + 5$$
$$3x - 6 + 2x - 6 + x - 4 = 3x + 5$$
$$3x + 2x + x - 3x = 5 + 6 + 6 + 4$$
$$3x = 21$$
$$\therefore x = 7.$$

24. $x + 1 + x + 2 + x + 4 = 2x + 12.$

$$x + 1 + x + 2 + x + 4 = 2x + 12$$
$$x + x + x - 2x = 12 - 1 - 2 - 4$$
$$\therefore x = 5.$$

25. $(2x-5)-(x-4)+(x-3)=x-4.$

$$\begin{aligned}(2x-5)-(x-4)+(x-3)&=x-4\\ 2x-5-x+4+x-3&=x-4\\ 2x-x+x-x&=5-4+3-4\\ \therefore x&=0.\end{aligned}$$

26. $4-5x-(1-8x)=63-x.$

$$\begin{aligned}4-5x-(1-8x)&=63-x\\ 4-5x-1+8x&=63-x\\ -5x+8x+x&=63-4+1\\ 4x&=60\\ \therefore x&=15.\end{aligned}$$

27. $3x-(x+10)-(x-3)=14-x.$

$$\begin{aligned}3x-(x+10)-(x-3)&=14-x\\ 3x-x-10-x+3&=14-x\\ 3x-x-x+x&=14-10+3\\ 2x&=21\\ \therefore x&=10\tfrac{1}{2}.\end{aligned}$$

28. $x^2-2x-3=x^2-3x+1.$

$$\begin{aligned}x^2-2x-3&=x^2-3x+1\\ x^2-x^2-2x+3x&=1+3\\ \therefore x&=4.\end{aligned}$$

29. $(x^2-9)-(x^2-16)+x=10.$

$$\begin{aligned}(x^2-9)-(x^2-16)+x&=10\\ x^2-9-x^2+16+x&=10\\ x^2-x^2+x&=10+9-16\\ \therefore x&=3.\end{aligned}$$

30. $x^2+8x-(x^2-x-2)=5(x+3)+3.$

$$\begin{aligned}x^2+8x-(x^2-x-2)&=5(x+3)+3\\ x^2+8x-x^2+x+2&=5x+15+3\\ x^2-x^2+8x+x-5x&=15+3-2\\ 4x&=16\\ \therefore x&=4.\end{aligned}$$

31. $x^2+x-2+x^2+2x-3=2x^2-7x-1.$

$$\begin{aligned}x^2+x-2+x^2+2x-3&=2x^2-7x-1\\ x^2+x^2-2x^2+x+2x+7x&=3+2-1\\ 10x&=4\\ \therefore x&=\tfrac{2}{5}.\end{aligned}$$

32. $$10x - (x - 5) = 2x + 47.$$

$$\begin{aligned} 10x - (x - 5) &= 2x + 47 \\ 10x - x + 5 &= 2x + 47 \\ 10x - x + 2x &= 47 - 5 \\ 7x &= 42 \\ \therefore x &= 6. \end{aligned}$$

33. $$7x - 5 - (6 - 8x) + 2 = 3x - 7 + 106.$$

$$\begin{aligned} 7x - 5 - (6 - 8x) + 2 &= 3x - 7 + 106 \\ 7x - 5 - 6 + 8x + 2 &= 3x - 7 + 106 \\ 7x + 8x - 3x &= 106 - 7 + 5 + 6 - 2 \\ 12x &= 108 \\ \therefore x &= 9. \end{aligned}$$

34. $$6x + 3 - (3x + 2) = (2x - 1) + 9.$$

$$\begin{aligned} 6x + 3 - (3x + 2) &= (2x - 1) + 9 \\ 6x + 3 - 3x - 2 &= 2x - 1 + 9 \\ 6x - 3x - 2x &= 9 - 1 - 3 + 2 \\ \therefore x &= 7. \end{aligned}$$

35. $$3(x + 10) + 4(x + 20) + 5x - 170 = 15 - 3x.$$

$$\begin{aligned} 3(x + 10) + 4(x + 20) + 5x - 170 &= 15 - 3x \\ 3x + 30 + 4x + 80 + 5x - 170 &= 15 - 3x \\ 3x + 4x + 5x + 3x &= 15 + 170 - 30 - 80 \\ 15x &= 75 \\ \therefore x &= 5. \end{aligned}$$

36. $$20 - x + 4(x - 1) - (x - 2) = 30.$$

$$\begin{aligned} 20 - x + 3(x - 1) - (x - 2) &= 30 \\ 20 - x + 4x - 4 - x + 2 &= 30 \\ -x + 4x - x &= 30 - 20 + 4 - 2 \\ 2x &= 12 \\ \therefore x &= 6. \end{aligned}$$

37. $$5x + 3 - (2x - 2) + (1 - x) = 6(9 - x).$$

$$\begin{aligned} 5x + 3 - (2x - 2) + (1 - x) &= 6(9 - x) \\ 5x + 3 - 2x + 2 + 1 - x &= 54 - 6x \\ 5x - 2x - x + 6x &= 54 - 3 - 2 - 1 \\ 8x &= 48 \\ \therefore x &= 6. \end{aligned}$$

Exercise 10. Page 27.

NOTE. — The teacher must require the pupils to give the *full* statement of the following problems as they are given in the book. The habit of making the full statement of a problem in algebraic language should be carefully cultivated when the problems are easy. Persons thus trained will ordinarily find no hard problems anywhere.

1. If a number is multiplied by 9, the product is 270. Find the number.

Let $x =$ the number.
Then $9x =$ the product.
But $270 =$ the product.
$\therefore 9x = 270.$
$\therefore x = 30.$

2. If the sum of the ages of a father and son is 60 years, and the father is 5 times as old as the son, what is the age of each?

Let $x =$ the number of years in the son's age.
Then $5x =$ the number of years in the father's age,
and $x + 5x =$ the number of years in both.
But $60 =$ the number of years in both.
$\therefore x + 5x = 60$
$6x = 60.$
$\therefore x = 10$, son's age,
$5x = 50$, father's age.

3. The sum of two numbers is 91, and the greater is 6 times the less. Find the numbers.

Let $x =$ the less number.
Then $6x =$ the greater number.
Therefore $x + 6x =$ the sum.
But $91 =$ the sum.
$\therefore x + 6x = 91$
$7x = 91.$
$\therefore x = 13$, less number,
$6x = 78$, greater number.

4. A tree 90 feet high was broken so that the part broken off was 8 times the length of the part kept standing. Find the length of each part.

Let $x =$ the number of feet in part left standing.
Then $8x =$ the number of feet in part broken off.
Therefore $x + 8x =$ the number of feet in both parts.
But $90 =$ the number of feet in both parts.

$$\therefore 9x = 90.$$
$$\therefore x = 10 \text{ feet, part left standing,}$$
$$8x = 80 \text{ feet, part broken off.}$$

5. The difference of two numbers is 7, and their sum is 53. Find the numbers.

Let $x =$ the smaller number.
Then $x + 7 =$ the greater number.
Therefore $x + x + 7 =$ the sum.
But $53 =$ the sum.

$$\therefore x + x + 7 = 53$$
$$x + x = 53 - 7$$
$$2x = 46.$$
$$\therefore x = 23, \text{ smaller number,}$$
$$23 + 7 = 30, \text{ greater number.}$$

6. The difference of two numbers is 12, and their sum is 84. Find the numbers.

Let $x =$ the smaller number.
Then $x + 12 =$ the greater number.
Therefore $x + x + 12 =$ the sum.
But $84 =$ the sum.

$$\therefore x + x + 12 = 84$$
$$x + x = 84 - 12.$$
$$2x = 72.$$
$$\therefore x = 36, \text{ smaller number,}$$
$$x + 12 = 48, \text{ greater number.}$$

7. Divide 35 into two parts so that one part shall be greater by 5 than the other part.

Let $x =$ the smaller part.
Then $x + 5 =$ the greater part.
Therefore $x + x + 5 =$ the sum.

But $35 =$ the sum.

$$\therefore x + x + 5 = 35$$
$$x + x = 35 - 5$$
$$2x = 30.$$
$$\therefore x = 15, \text{ smaller part,}$$
$$x + 5 = 20, \text{ greater part.}$$

8. Three times a given number is equal to the number increased by 40. Find the number.

Let $x =$ the number.
Then $3x =$ three times the number,
and $x + 40 =$ the number increased by 40.
But these two expressions are equal.

$$\therefore 3x = x + 40$$
$$3x - x = 40$$
$$2x = 40.$$
$$\therefore x = 20.$$

9. Three times a given number diminished by 24 is equal to the given number. Find the number.

Let $x =$ the number.
Then $3x - 24 =$ three times the number diminished by 24.
But these two expressions are equal.

$$\therefore 3x - 24 = x$$
$$3x - x = 24$$
$$2x = 24.$$
$$\therefore x = 12.$$

10. One number is 4 times another, and their difference is 30. Find the numbers.

Let $x =$ the smaller number.
Then $4x =$ the greater number.
Therefore $4x - x =$ their difference.
But $30 =$ their difference.

$$\therefore 4x - x = 30$$
$$3x = 30.$$
$$\therefore x = 10, \text{ smaller number,}$$
$$4x = 40, \text{ greater number.}$$

11. The sum of two numbers is 36, and one of them exceeds twice the other by 6. Find the numbers.

Let $x =$ the smaller number.
Then $2x + 6 =$ the greater number.
Therefore $x + 2x + 6 =$ their sum.
But $36 =$ their sum.

$$\therefore x + 2x + 6 = 36$$
$$x + 2x = 36 - 6$$
$$3x = 30$$
$$\therefore x = 10, \text{ smaller number,}$$
$$2x + 6 = 26, \text{ greater number.}$$

12. The sum of two numbers is 40, and 5 times the smaller exceeds 2 times the greater by 25. Find the numbers.

Let $x =$ the greater number.
Then $40 - x =$ the smaller number.
Therefore $5(40 - x) - 2x =$ the excess.
But $25 =$ the excess.

$$\therefore 25 = 5(40 - x) - 2x$$
$$25 = 200 - 5x - 2x$$
$$5x + 2x = 200 - 25$$
$$7x = 175.$$
$$\therefore x = 25, \text{ greater number,}$$
$$40 - x = 15, \text{ smaller number.}$$

13. Divide 30 into two parts such that 4 times the greater part exceeds 5 times the smaller part by 30. Find the parts.

Let $x =$ the smaller part.
Then $30 - x =$ the greater part.
Therefore $4(30 - x) - 5x =$ the excess.
But $30 =$ the excess.

$$\therefore 30 = 4(30 - x) - 5x$$
$$30 = 120 - 4x - 5x$$
$$4x + 5x = 120 - 30$$
$$9x = 90.$$
$$\therefore x = 10, \text{ smaller part,}$$
$$30 - x = 20, \text{ greater part.}$$

14. The sum of two numbers is 27, and twice the greater number increased by 3 times the less is 61. Find the numbers.

Let $x =$ the smaller number.
Then $27 - x =$ the greater number.
Therefore $2(27 - x) + 3x =$ the sum.
But $61 =$ the sum.

$$\therefore 2(27 - x) + 3x = 61$$
$$54 - 2x + 3x = 61$$
$$3x - 2x = 61 - 54.$$
$$\therefore x = 7, \text{ smaller number,}$$
$$27 - x = 20, \text{ greater number.}$$

15. The sum of two numbers is 32, and five times the smaller is 3 times the greater number. Find the numbers.

Let $x =$ the smaller number.
Then $32 - x =$ the greater number.
Therefore $5x =$ five times the smaller,
and $3(32 - x) =$ three times the greater.
But these two expressions are equal.

$$\therefore 5x = 3(32 - x)$$
$$5x = 96 - 3x$$
$$5x + 3x = 96$$
$$8x = 96.$$
$$\therefore x = 12, \text{ smaller number,}$$
$$32 - x = 20, \text{ greater number.}$$

Exercise 11. Page 29.

1. A farmer sold a horse and a cow for \$210. He sold the horse for four times as much as the cow. How much did he get for each?

Let $x =$ the price of the cow in dollars.
Then $4x =$ the price of the horse in dollars.
Therefore $x + 4x =$ the price of both in dollars.
But $210 =$ the price of both in dollars.

$$\therefore x + 4x = 210$$
$$5x = 210.$$
$$\therefore x = 42, \text{ price of the cow,}$$
$$4x = 168, \text{ price of the horse.}$$

2. Three times the excess of a certain number over 6 is equal to the number plus 144. Find the number.

Let $x =$ the number.
Then $x - 6 =$ the excess of the number over 6,
and $3(x-6) =$ three times the excess of the number over 6,
and $x + 144 =$ the number increased by 144.

But these two expressions are equal.

$$\therefore 3(x-6) = x + 144$$
$$3x - 18 = x + 144$$
$$3x - x = 144 + 18$$
$$2x = 162.$$
$$\therefore x = 81.$$

3. Thirty-one times a certain number is as much above 40 as nine times the number is below 40. Find the number.

Let $x =$ the number.
Then $31x - 40 =$ the excess of thirty-one times the number over 40,
and $40 - 9x =$ the excess of 40 over nine times the number.

But these two expressions are equal.

$$\therefore 31x - 40 = 40 - 9x$$
$$31x + 9x = 40 + 40$$
$$40x = 80.$$
$$\therefore x = 2.$$

4. Two numbers differ by 10, and their sum is equal to seven times their difference. Find the numbers.

Let $x =$ the smaller number.
Then $x + 10 =$ the greater number.
Therefore $x + x + 10 =$ their sum,
and $7 \times 10 =$ seven times their difference.

But these two expressions are equal.

$$\therefore x + x + 10 = 7 \times 10$$
$$x + x + 10 = 70$$
$$2x = 70 - 10$$
$$2x = 60.$$
$$\therefore x = 30, \text{ smaller number,}$$
$$x + 10 = 40, \text{ greater number.}$$

5. Find three consecutive numbers, x, $x+1$, and $x+2$, whose sum is 78.

Let $x =$ the first number,

$x + 1 =$ the second number,

and $x + 2 =$ the third number.

Hence $x + x + 1 + x + 2 =$ their sum.

But $78 =$ their sum.

$$\therefore x + x + 1 + x + 2 = 78$$
$$x + x + x = 78 - 1 - 2$$
$$3x = 75.$$
$$\therefore x = 25, \text{ first number,}$$
$$x + 1 = 26, \text{ second number,}$$
$$x + 2 = 27, \text{ third number.}$$

6. Find five consecutive numbers whose sum is 35.

Let $x =$ the first number,

$x + 1 =$ the second number,

$x + 2 =$ the third number,

$x + 3 =$ the fourth number,

$x + 4 =$ the fifth number.

Then $x + x + 1 + x + 2 + x + 3 + x + 4$ $=$ their sum.

But $35 =$ their sum.

$$\therefore x + x + 1 + x + 2 + x + 3 + x + 4 = 35$$
$$x + x + x + x + x = 35 - 1 - 2 - 3 - 4$$
$$5x = 25.$$
$$\therefore x = 5, \text{ first number,}$$
$$x + 1 = 6, \text{ second number,}$$
$$x + 2 = 7, \text{ third number,}$$
$$x + 3 = 8, \text{ fourth number,}$$
$$x + 4 = 9, \text{ fifth number.}$$

7. The sum of the ages of A and B is 40 years, and 10 years hence A will be twice as old as B. Find their present ages.

Let $x =$ the number of years in A's age.

Then $40 - x =$ the number of years in B's age,

$x + 10 =$ the number of years in A's age 10 years hence,

and $\quad 2(40-x+10)=$ twice number of years in B's age 10 years hence.

But these two expressions are equal.

$$\therefore x+10=2(40-x+10)$$
$$x+10=2(50-x)$$
$$x+10=100-2x$$
$$x+2x=100-10$$
$$3x=90.$$
$$\therefore x=30, \text{ A's age,}$$
$$40-x=10, \text{ B's age.}$$

8. A father is four times as old as his son, and in 5 years he will be only three times as old. Find their present ages.

Let $\quad x=$ the number of years in son's age.

Then $\quad 4x=$ the number of years in father's age,

Hence $\quad 3(x+5)=$ three times the number of years in son's age in 5 years,

and $\quad 4x+5=$ the number of years in father's age in 5 years.

But these two expressions are equal.

$$\therefore 4x+5=3(x+5)$$
$$4x+5=3x+15$$
$$4x-3x=15-5.$$
$$\therefore x=10, \text{ son's age,}$$
$$4x=40, \text{ father's age.}$$

9. One man is 60 years old, and another man is 50 years. How many years ago was the first man twice as old as the second?

Let $\quad x=$ the number of years ago one was twice the age of the other.

Then $\quad 60-x=$ the age of the first at that time,

and $\quad 2(50-x)=$ twice the age of the other at that time.

But these two expressions are equal.

$$\therefore 60-x=2(50-x)$$
$$60-x=100-2x$$
$$2x-x=100-60.$$
$$\therefore x=40.$$

10. A man 50 years old has a son 10 years old. In how many years will the father be three times as old as the son?

Let $x =$ the number of years when the father's age will be three times that of son's.

Then $50 + x =$ the father's age at that time,

and $3(10 + x) =$ three times the son's age at that time.

But these two expressions are equal.

$$\therefore 3(10 + x) = 50 + x$$
$$30 + 3x = 50 + x$$
$$3x - x = 50 - 30$$
$$2x = 20.$$
$$\therefore x = 10.$$

11. A has \$100, and B has \$20. How much must A give B in order that they may each have the same sum?

Let $x =$ the number of dollars A must give B.

Then $100 - x =$ the number of dollars A will have,

and $20 + x =$ the number of dollars B will have.

But these amounts are equal.

$$\therefore 20 + x = 100 - x$$
$$x + x = 100 - 20$$
$$2x = 80.$$
$$\therefore x = 40.$$

Therefore A must give B \$40.

12. A banker paid \$63 in 5-dollar bills and 2-dollar bills. He paid just as many 5-dollar bills as 2-dollar bills. How many bills of each kind did he pay?

Let $x =$ the number of bills of each kind.

Then $2x =$ the value of the 2-dollar bills,

and $5x =$ the value of the 5-dollar bills.

Therefore $2x + 5x =$ the sum.

But $63 =$ the sum.

$$\therefore 2x + 5x = 63$$
$$7x = 63.$$
$$\therefore x = 9.$$

Exercise 12. Page 30.

1. In a company of 90 persons, composed of men, women, and children, there are three times as many children as men, and twice as many women as men. How many are there of each?

Let $x =$ the number of men.
Then $2x =$ the number of women,
and $3x =$ the number of children.
Therefore $x + 2x + 3x =$ the whole number.
But $90 =$ the whole number.

$$\therefore x + 2x + 3x = 90$$
$$6x = 90.$$
$$\therefore x = 15, \text{ number of men,}$$
$$2x = 30, \text{ number of women,}$$
$$3x = 45, \text{ number of children.}$$

2. Find the number whose double exceeds 70 by as much as the number itself is less than 80.

Let $x =$ the number.
Then $2x - 70 =$ the excess of its double over 70,
and $80 - x =$ the excess of 80 over the number.
But these two expressions are equal.

$$\therefore 2x - 70 = 80 - x$$
$$2x + x = 80 + 70$$
$$3x = 150.$$
$$\therefore x = 50.$$

3. A farmer employed two men to build 112 rods of wall. One of them built on the average 4 rods a day, and the other 3 rods a day. How many days did they work?

Let $x =$ the number of days they worked.
Then $x(4 + 3) =$ the number of rods they built.
But $112 =$ the number of rods they built.

$$\therefore x(4 + 3) = 112$$
$$7x = 112.$$
$$\therefore x = 16.$$

4. Two men travel in *opposite* directions, one 30 miles a day, and the other 20 miles a day. In how many days will they be 350 miles apart?

Let $x =$ the number of days.
Then $x(30 + 20) =$ the number of miles they will be apart.
But $350 =$ the number of miles they will be apart.

$$\therefore x(30 + 20) = 350$$
$$50x = 350.$$
$$\therefore x = 7.$$

3. Two men travel in the same direction, one 30 miles a day, and the other 20 miles a day. In how many days will they be 350 miles apart?

Let $x =$ the number of days.

Then $x(30 - 20) =$ the number of miles they will be apart.

But $350 =$ the number of miles they will be apart.

$$\therefore x(30 - 20) = 350$$

$$10x = 350.$$

$$\therefore x = 35.$$

4. A man bought 3 equal lots of hay for $480. For the first lot he gave $26 a ton, for the second $16, for the third $18. How many tons did he buy in all?

Let $x =$ the number of tons in each lot.

Then $3x =$ the whole number of tons,

and $26x =$ the value in dollars of first lot,

$16x =$ the value in dollars of second lot,

$18x =$ the value in dollars of third lot.

Then $26x + 16x + 18x =$ the value in dollars of all.

But $480 =$ the value in dollars of all.

$$\therefore 26x + 16x + 18x = 480$$

$$60x = 480.$$

$$\therefore x = 8,$$

and $3x = 24.$

5. A man sold a quantity of wood for $84, one half of it at $3 a cord, and the other half at $4 a cord. How many cords did he sell?

Let $x =$ the number of cords.

Then $\frac{3x}{2} =$ the value in dollars of one half,

and $\frac{4x}{2} =$ the value in dollars of the other half,

$\frac{3x}{2} + \frac{4x}{2} =$ the value in dollars of the whole.

But $84 =$ the value in dollars of the whole.

$$\therefore \frac{3x}{2} + \frac{4x}{2} = 84$$

$$3x + 4x = 168$$

$$7x = 168$$

$$\therefore x = 24.$$

8. If $2x - 3$ stands for 29, for what number will $4 + x$ stand?

If $\qquad 2x - 3 = 29$

$$2x = 29 + 3$$
$$2x = 32.$$
$$\therefore x = 16,$$

and $\qquad 4 + x = 20.$

9. At an election two opposing candidates received together 2044 votes, and one received 104 more votes than the other. How many votes did each candidate receive?

Let $\qquad x =$ the number of votes one received.
Then $\qquad x + 104 =$ the number of votes the other received.
Therefore $\qquad x + x + 104 =$ the number of votes in all.
But $\qquad 2044 =$ the number of votes in all.

$$\therefore x + x + 104 = 2044$$
$$x + x = 2044 - 104$$
$$2x = 1940.$$
$$\therefore x = 970,$$

and $\qquad x + 104 = 1074.$

Exercise 13. Page 31.

1. A man walks 4 miles an hour for x hours, and another man walks 3 miles an hour for $x + 2$ hours. If they each walk the same distance, how many miles does each walk?

Let $\qquad 4x =$ the number of miles first walks.
Then $\qquad 3(x + 2) =$ the number of miles second walks.
But these two expressions are equal.

$$\therefore 4x = 3(x + 2)$$
$$4x = 3x + 6$$
$$4x - 3x = 6.$$
$$\therefore x = 6$$
$$4x = 24, \text{ number of miles each walks.}$$

2. A has twice as much money as B; but if A gives B $30, it will take twice as much as A has left to equal B's. How much money has each?

Let $x =$ the number of dollars B has.

Then $2x =$ the number of dollars A has.

Therefore $2(2x - 30) =$ twice the number of dollars A has left,

and $x + 30 =$ number of dollars B has after the gift.

But these two expressions are equal.

$$\therefore 2(2x - 30) = x + 30$$
$$4x - 60 = x + 30$$
$$4x - x = 30 + 60$$
$$3x = 90.$$
$$\therefore x = 30,$$

and $2x = 60.$

Therefore A has \$60, and B \$30.

3. I have \$12.75 in two-dollar bills and twenty-five cent pieces, and I have twice as many bills as twenty-five cent pieces. How many have I of each?

Let $x =$ the number of quarters.

Then $2x =$ the number of bills.

Therefore $25x =$ the number of cents in quarters,

and $200 \times 2x$ or $400x =$ the number of cents in bills.

Therefore $25x + 400x =$ the number of cents in all.

But $1275 =$ the number of cents in all.

$$\therefore 25x + 400x = 1275$$
$$425x = 1275.$$

$\therefore x = 3$, number of quarters,

$2x = 6$, number of bills.

4. I have in mind a certain number. If this number is diminished by 8 and the remainder multiplied by 8, the result is the same as if the number was diminished by 6 and the remainder multiplied by 6. What is the number?

Let $x =$ the number.

Then $8(x - 8) =$ eight times the number diminished by 8,

and $6(x - 6) =$ six times the number diminished by 6.

But these two expressions are equal.

$$\therefore 8(x - 8) = 6(x - 6)$$
$$8x - 64 = 6x - 36$$
$$8x - 6x = 64 - 36$$
$$2x = 28.$$
$$\therefore x = 14.$$

5. I have five times as many half-dollars as quarters, and the half-dollars and quarters amount to \$11. How many of each have I?

Let $x =$ the number of quarters.

Then $5x =$ the number of half-dollars.

Therefore $25x =$ the number of cents in quarters,

and $50 \times 5x = 250x$, the number of cents in half-dollars.

Therefore $25x + 250x =$ the number of cents in all.

But $1100 =$ the number of cents in all.

$$\therefore 25x + 250x = 1100$$
$$275x = 1100.$$
$$\therefore x = 4, \text{ number of quarters,}$$

and $5x = 20$, number of half-dollars.

6. A man pays a debt of \$91 with ten-dollar bills and one-dollar bills, paying three times as many one-dollar bills as ten-dollar bills. How many bills of each kind does he pay?

Let $x =$ the number of ten-dollar bills.

Then $3x =$ the number of one-dollar bills.

Therefore $10x =$ the value of the ten-dollar bills in dollars,

and $3x =$ the value of the one-dollar bills in dollars.

Therefore $10x + 3x =$ the amount in dollars.

But $91 =$ the amount in dollars.

$$\therefore 10x + 3x = 91$$
$$13x = 91.$$
$$\therefore x = 7, \text{ number of ten-dollar bills,}$$

and $3x = 21$, number of one-dollar billls.

7. A father is four times as old as his son, but 4 years hence he will be only three times as old as his son. How old is each?

Let $x =$ the number of years in son's age.

Then $4x =$ the number of years in father's age.

Therefore $3(x + 4) =$ three times the number of years in son's age in 4 years,

and $4x + 4 =$ the number of years in father's age in 4 years.

But these two expressions are equal.

$$\therefore 4x + 4 = 3(x + 4)$$
$$4x + 4 = 3x + 12$$
$$4x - 3x = 12 - 4.$$
$$\therefore x = 8, \text{ son's age,}$$
$$4x = 32, \text{ father's age.}$$

8. A workman was employed for 24 days. For every day he worked he was to receive \$1.50, and for every day he was idle he was to pay \$0.50 for his board. At the end of the time he received \$28. How many days did he work?

Let $x =$ the number of days he worked.
Then $24 - x =$ the number of days he was idle.
Therefore $150\,x =$ the number of cents he received,
and $50\,(24 - x) =$ the number of cents he paid.
Now $50\,(24 - x) = 1200 - 5\,x$.
Hence $150\,x - (1200 - 50\,x) =$ the number of cents he had left.
But $2800 =$ the number of cents he had left.

$$\therefore 150\,x - 1200 + 50\,x = 2800$$
$$150\,x + 50\,x = 2800 + 1200$$
$$200\,x = 4000.$$
$\therefore x = 20$, number of days he worked.

Exercise 14. Page 32.

1. A boy has 4 hours at his disposal. How far can he ride into the country at the rate of 9 miles an hour and walk back at the rate of 3 miles an hour, if he returns just on time?

Let $x =$ the number of hours he can ride.
Then $4 - x =$ the number of hours he walks.
Therefore $9\,x =$ the number of miles he rides,
and $3\,(4 - x) =$ the number of miles he walks.
But these two expressions are equal.

$$\therefore 9\,x = 3\,(4 - x)$$
$$9\,x = 12 - 3\,x$$
$$9\,x + 3\,x = 12$$
$$12\,x = 12$$
$\therefore x = 1$, number of hours he can ride,
$9\,x = 9$, number of miles.

2. A has \$180, and B has \$80. How much must A give B in order that 6 times B's money shall be equal to 7 times A's.

Let $x =$ the number of dollars A gives B.
Then $7\,(180 - x) =$ seven times the number of dollars A has left,

and $6(80 + x) =$ six times the number of dollars B has.

But these two expressions are equal.

$$\therefore 6(80 + x) = 7(180 - x)$$
$$480 + 6x = 1260 - 7x$$
$$6x + 7x = 1260 - 480$$
$$13x = 780.$$
$$\therefore x = 60.$$

Therefore A must give $60 to B.

3. A grocer has two kinds of tea, one kind worth 45 cents a pound, and the other worth 65 cents a pound. How many pounds of each kind must he take to make 80 pounds, worth 50 cents a pound?

Let $x =$ the number of pounds of 65-cent tea.
Then $80 - x =$ the number of pounds of 45-cent tea.
Therefore $65x =$ the value in cents of 65-cent tea,
and $45(80 - x) =$ the value in cents of 45-cent tea.
Hence $65x + 45(80 - x) =$ the value in cents of the whole.
But 80×50, or $4000 =$ the value in cents of the whole.

$$\therefore 65x + 45(80 - x) = 4000$$
$$65x + 3600 - 45x = 4000$$
$$65x - 45x = 4000 - 3600$$
$$20x = 400.$$

$\therefore x = 20$, number of pounds of 65-cent tea,
$80 - x = 60$, number of pounds of 45-cent tea.

4. A tank holding 1200 gallons has three pipes. The first lets in 8 gallons a minute, the second 10 gallons, and the third 12 gallons a minute. In how many minutes will the tank be filled?

Let $x =$ the number of minutes.
Then $8x =$ the number of gallons the first lets in,
$10x =$ the number of gallons the second lets in,
and $12x =$ the number of gallons the third lets in.
Hence $8x + 10x + 12x =$ the number of gallons all let in.
But $1200 =$ the number of gallons all let in.

$$\therefore 8x + 10x + 12x = 1200$$
$$30x = 1200.$$
$$\therefore x = 40.$$

5. The fore and hind wheels of a carriage are 10 feet and 12 feet respectively in circumference. How many feet will the carriage have

passed over when the fore wheel has made 250 revolutions more than the hind wheel?

Let $x =$ the number of revolutions of fore wheel.

Then $x - 250 =$ the number of revolutions of hind wheel.

Therefore $10x =$ the number of feet fore wheel passes over,

and $12(x - 250) =$ the number of feet hind wheel passes over.

But these two expressions are equal.

$$\therefore 12(x - 250) = 10x$$
$$12x - 3000 = 10x$$
$$12x - 10x = 3000$$
$$2x = 3000.$$

$\therefore x = 1500$, number of revolutions of fore wheel.

Therefore $10 \times 1500 = 15{,}000$, number of feet passed over.

6. Divide a yard of tape into two parts so that one part shall be 6 inches longer than the other part.

Let $x =$ the number of inches in one part.

Then $x + 6 =$ the number of inches in other part.

Therefore $x + x + 6 =$ the number of inches in all.

But $36 =$ the number of inches in all.

$$\therefore x + x + 6 = 36$$
$$x + x = 36 - 6$$
$$2x = 30.$$

$\therefore x = 15$, number of inches in one part,

$x + 6 = 21$, number of inches in the other part.

7. A boy bought 7 dozen oranges for \$1.50. For a part he paid 20 cents a dozen; and for the remainder, 25 cents a dozen. How many dozens of each kind did he buy?

Let $x =$ the number of dozen at 25 cents.

Then $7 - x =$ the number of dozen at 20 cents,

$25x =$ the number of cents paid for oranges at 25 cents a dozen,

and $20(7 - x) =$ the number of cents paid for oranges at 20 cents a dozen.

Hence $25x + 20(7 - x) =$ the number of cents paid in all.

But $150 =$ the number of cents paid in all.

$$\therefore 25x + 20(7 - x) = 150$$
$$25x + 140 - 20x = 150$$
$$25x - 20x = 150 - 140$$
$$5x = 10.$$
$\therefore x = 2$, number of dozen at 25 cents,
$7 - x = 5$, number of dozen at 20 cents.

8. How can a bill of \$3.30 be paid in quarters and ten-cent pieces so as to pay three times as many ten-cent pieces as quarters?

Let $x =$ the number of quarters.
Then $3x =$ the number of ten-cent pieces.
Therefore $25x =$ the number of cents in quarters,
and $10 \times 3x = 30x$, the number of cents in ten-cent pieces.
Therefore $25x + 30x =$ the number of cents in all.
But $330 =$ the number of cents in all.

$$\therefore 25x + 30x = 330$$
$$55x = 330.$$
$\therefore x = 6$, number of quarters,
$3x = 18$, number of ten-cent pieces.

Exercise 15. Page 39.

Find the sum of:

1. $5c$, $23c$, c, $11c$.

Find the sum of $5c$, $23c$, c, $11c$.
The sum of the coefficients is $5 + 23 + 1 + 11 = 40$.
Hence the sum of the numbers is $40c$.

2. $4a$, $3a$, $7a$, $10a$.

Find the sum of $4a$, $3a$, $7a$, $10a$.
The sum of the coefficients is $4 + 3 + 7 + 10 = 24$.
Hence the sum of the numbers is $24a$.

3. $7x$, $12x$, $11x$, $9x$.

Find the sum of $7x$, $12x$, $11x$, $9x$.
The sum of the coefficients is $7 + 12 + 11 + 9 = 39$.
Hence the sum of the numbers is $39x$.

4. $6y,\ 8y,\ 2y,\ 35y.$

Find the sum of $6y,\ 8y,\ 2y,\ 35y$.
The sum of the coefficients is $6 + 8 + 2 + 35 = 51$.
Hence the sum of the numbers is $51y$.

5. $-3a,\ -5a,\ -18a.$

Find the sum of $-3a,\ -5a,\ -18a$.
The sum of the coefficients is $-3 - 5 - 18 = -26$.
Hence the sum of the numbers is $-26a$.

6. $-5x,\ -6x,\ -18x,\ -11x.$

Find the sum of $-5x,\ -6x,\ -18x,\ -11x$.
The sum of the coefficients is $-5 - 6 - 18 - 11 = -40$.
Hence the sum of the numbers is $-40x$.

7. $-3b,\ -b,\ -9b,\ -4b.$

Find the sum of $-3b,\ -b,\ -9b,\ -4b$.
The sum of the coefficients is $-3 - 1 - 9 - 4 = -17$.
Hence the sum of the numbers is $-17b$.

8. $-z,\ -2z,\ -10z,\ -53z.$

Find the sum of $-z,\ -2z,\ -10z,\ -53z$.
The sum of the coefficients is $-1 - 2 - 10 - 53 = -66$.
Hence the sum of the numbers is $-66z$.

9. $-11m,\ -3m,\ -5m,\ -m.$

Find the sum of $-11m,\ -3m,\ -5m,\ -m$.
The sum of the coefficients is $-11 - 3 - 5 - 1 = -20$.
Hence the sum of the numbers is $-20m$.

10. $5d,\ -d,\ -4d,\ 2d.$

Find the sum of $5d,\ -d,\ -4d,\ 2d$.
The sum of the positive coefficients is $5 + 2 = 7$.
The sum of the negative coefficients is $-1 - 4 = -5$.
The difference between 7 and 5 is 2, and the sign of the greater is positive.
Hence the required sum is $2d$.

11. $13n,\ 13n,\ -11n,\ -6n,\ -9n,\ n,\ 2n,\ -3n.$

Find the sum of $13n,\ 13n,\ -11n,\ -6n,\ -9n,\ n,\ 2n,\ -3n$.
The sum of the positive coefficients is $13 + 13 + 1 + 2 = 29$.
The sum of the negative coefficients is $-11 - 6 - 9 - 3 = -29$.
The difference between 29 and 29 is 0.
Hence the required sum is 0.

12. $5g, -3g, -6g, -4g, 20g, -5g, -11g, -14g.$

Find the sum of $5g, -3g, -6g, -4g, 20g, -5g, -11g, -14g$.

The sum of the positive coefficients is $5 + 20 = 25$.

The sum of the negative coefficients is $-3-6-4-5-11-14 = -43$.

The difference between 25 and 43 is 18, and the sign of the greater is negative.

Hence the required sum is $-18g$.

13. $-9a^2, 5a^2, 6a^2, a^2, 2a^2, -a^2, -3a^2.$

Find the sum of $-9a^2, 5a^2, 6a^2, a^2, 2a^2, -a^2, -3a^2$.

The sum of the positive coefficients is $5 + 6 + 1 + 2 = 14$.

The sum of the negative coefficients is $-9-1-3 = -13$.

The difference between 14 and 13 is 1, and the sign of the greater is positive.

Hence the required sum is a^2.

14. $3x^3, -2x^3, -5x^3, -7x^3, -x^3, 2x^3, -10x^3, -x^3.$

Find the sum of $3x^3, -2x^3, -5x^3, -7x^3, -x^3, 2x^3, -10x^3, -x^3$.

The sum of the positive coefficients is $3 + 2 = 5$.

The sum of the negative coefficients is $-2-5-7-1-10-1 = -26$.

The difference between 5 and 26 is 21, and the sign of the greater is negative.

Hence the required sum is $-21x^3$.

15. $4a^2b^2, -a^2b^2, -6a^2b^2, 4a^2b^2, -2a^2b^2, a^2b^2.$

Find the sum of $4a^2b^2, -a^2b^2, -6a^2b^2, 4a^2b^2, -2a^2b^2, a^2b^2$.

The sum of the positive coefficients is $4 + 4 + 1 = 9$.

The sum of the negative coefficients is $-1-6-2 = -9$.

The difference between 9 and 9 is 0.

Hence the required sum is 0.

16. $6mn, -5mn, mn, -3mn, 4mn.$

Find the sum of $6mn, -5mn, mn, -3mn, 4mn$.

The sum of the positive coefficients is $6 + 1 + 4 = 11$.

The sum of the negative coefficients is $-5-3 = -8$.

The difference between 11 and 8 is 3, and the sign of the greater is positive.

Hence the required sum is $3mn$.

17. $3\,xyz,\ -2\,xyz,\ 5\,xyz,\ -7\,xyz,\ xyz.$

Find the sum of $3\,xyz,\ -2\,xyz,\ 5\,xyz,\ -7\,xyz,\ xyz.$
The sum of the positive coefficients is $3+5+1=9$.
The sum of the negative coefficients is $-2-7=-9$.
The difference between 9 and 9 is 0.
Hence the required sum is 0.

18. $5\,a^3b^3c^3,\ -7\,a^3b^3c^3,\ -3\,a^3b^3c^3,\ 2\,a^3b^3c^3.$

Find the sum of $5\,a^3b^3c^3,\ -7\,a^3b^3c^3,\ -3\,a^3b^3c^3,\ 2\,a^3b^3c^3.$
The sum of the positive coefficients is $5+2=7$.
The sum of the negative coefficients is $-7-3=-10$.
The difference between 7 and 10 is 3, and the sign of the greater is negative.
Hence the required sum is $-3a^3b^3c^3$.

19. $11\,abcd,\ -10\,abcd,\ -9\,abcd,\ -abcd.$

Find the sum of $11\,abcd,\ -10\,abcd,\ -9\,abcd,\ -abcd.$
The sum of the positive coefficients is 11.
The sum of the negative coefficients is $-10-9-1=-20$.
The difference between 20 and 11 is 9, and the sign of the greater is negative.
Hence the required sum is $-9\,abcd$.

20. Subtract $-a$ from $-b$, and find the value of the result if $a=-4,\ b=-5$.

$$-b-(-a)=-b+a=5-4=1.$$

When $a=4,\ b=-2,\ c=-3$, find the difference in the values of:

21. $a-b+c$ and $-a+b+c$.

$$a-b+c=4+2-3=3$$
$$-a+b+c=-4-2-3=-9$$
$$3-(-9)=3+9=12.$$

22. $a+(-b)+c$ and $a-(-b)+c$.

$$a+(-b)+c=a-b+c=4+2-3=3$$
$$a-(-b)+c=a+b+c=4-2-3=-1$$
$$3-(-1)=3+1=4.$$

23. $-a-(-b)+c$ and $-(-a)+(-b)-c$.

$$-a-(-b)+c=-a+b+c=-4-2-3=-9$$
$$-(-a)+(-b)-c=+a-b-c=4+2+3=9$$
$$-9-(+9)=-9-9=-18.$$

24. $a - b + (-c)$ and $a - (-b) - (-c)$.

$a - b + (-c) = a - b - c = 4 + 2 + 3 = 9$
$a - (-b) - (-c) = a + b + c = 4 - 2 - 3 = -1$
$9 - (-1) = 9 + 1 = 10.$

Exercise 16. Page 42.

Find the product of:

1. $5a^2$ and $6a^3$.

$5a^2 \times 6a^3 = 30a^5.$

2. $8ab$ and $5a^3b^2$.

$8ab \times 5a^3b^2 = 40a^4b^3.$

3. $9xy$ and $7xy$.

$9xy \times 7xy = 63x^2y^2.$

4. $2a^2b$ and $a^3b^4c^2$.

$2a^2b \times a^3b^4c^2 = 2a^5b^5c^2.$

5. $3a^3b^7c^8$ and $3a^4b^2c$.

$3a^3b^7c^8 \times 3a^4b^2c = 9a^7b^9c^9.$

6. $2a$ and $-5a$.

$2a \times -5a = -10a^2.$

7. $-3a$ and $-4b$.

$-3a \times -4b = 12ab.$

8. $-ab$ and a^3b^2.

$-ab \times a^3b^2 = -a^4b^3.$

9. $-2ab^4$ and $-5a^4bc$.

$-2ab^4 \times -5a^4bc = 10a^5b^5c.$

10. $-2x^6y^3z$ and $-6xy^2z$.

$-2x^6y^3z \times -6xy^2z = 12x^7y^5z^2.$

11. $3a^2b$, $-5ab^2$, and $-7a^4b^2$.

$3a^2b \times -5ab^2 \times -7a^4b^2 = 105a^7b^5.$

12. $2a^2bc^3$, $-3a^3b^2c$, and $-ab^2c^3$.

$2a^2bc^3 \times -3a^3b^2c \times -ab^2c^3 = 6a^6b^5c^7.$

13. $2\,b^2c^2x^2$, $2\,a^2b^2c^3$, and $-3\,a^3bx^3$.

$2\,b^2c^2x^2 \times 2\,a^2b^2c^3 \times -3\,a^3bx^3 = -12\,a^5b^5c^5x^5.$

14. $2\,a^3b^2c$, $-3\,a^2b^3c$, and $-4\,a^2bc^3$.

$2\,a^3b^2c \times -3\,a^2b^3c \times -4\,a^2bc^3 = 24\,a^7b^6c^5.$

15. $7\,am^2x^3$, $3\,a^4m^2x^3$, and $-2\,amx$.

$7\,am^2x^3 \times 3\,a^4m^2x^3 \times -2\,amx = -42\,a^6m^5x^7.$

16. $-3\,x^2y^2z^2$, $2\,x^2yz^3$, and $-5\,x^4yz$.

$$-3\,x^2y^2z^2 \times 2\,x^2yz^3 \times -5\,x^4yz = 30\,x^8y^4z^6.$$

If $a = -2$, $b = 3$, and $c = -1$, find the value of:

17. $2\,ab^2 - 3\,bc^2 + c$.

$$2\,ab^2 - 3\,bc^2 + c = (2 \times -2 \times 3 \times 3) - (3 \times 3 \times -1 \times -1) + (-1)$$
$$= -36 - 9 - 1 = -46.$$

18. $4\,a^2 - 2\,b^2 - c^2$.

$$4\,a^2 - 2\,b^2 - c^2 = (4 \times -2 \times -2) - (2 \times 3 \times 3) - (-1 \times -1)$$
$$= 16 - 18 - 1 = -3.$$

19. $5\,a + 2\,b - 4\,c^4$.

$$5\,a + 2\,b - 4\,c^4 = (5 \times -2) + (2 \times 3) - (4 \times -1 \times -1 \times -1 \times -1)$$
$$= -10 + 6 - 4 = -8.$$

20. $2\,a^3 - 3\,b + 8\,c^2$.

$$2\,a^3 - 3\,b + 8\,c^2 = (2 \times -2 \times -2 \times -2) - (3 \times 3) + (8 \times -1 \times -1)$$
$$= -16 - 9 + 8 = -17.$$

21. $-a + 3\,b - 2\,c^2$.

$$-a + 3\,b - 2\,c^2 = -(-2) + (3 \times 3) - (2 \times -1 \times -1)$$
$$= 2 + 9 - 2 = 9.$$

22. $-a^3 - 2\,b - 10\,c$.

$$-a^3 - 2\,b - 10\,c = -(-2 \times -2 \times -2) - (2 \times 3) - (10 \times -1)$$
$$= 8 - 6 + 10 = 12.$$

23. $3\,a^3 - 3\,b^3 - 3\,c^3$.

$$3\,a^3 - 3\,b^3 - 3\,c^3 = [3 \times (-2)^3] - [3 \times (3)^3] - [3 \times (-1)^3]$$
$$= -24 - 81 + 3 = -102.$$

24. $2ab^2 - 3bc^2 + 2ac$.

$2ab^2 - 3bc^2 + 2ac = (2 \times -2 \times 3 \times 3) - (3 \times 3 \times -1 \times -1)$
$+ (2 \times -2 \times -1) = -36 - 9 + 4 = -41$.

25. $3abc + 5a^2b^2 - 2a^2b$.

$3abc + 5a^2b^2 - 2a^2b = (3 \times -2 \times 3 \times -1) + (5 \times -2 \times -2 \times 3 \times 3)$
$- (2 \times -2 \times -2 \times 3) = 18 + 180 - 24 = 174$.

26. $ab^2c^2 + 2abc^2 + a^2b^2c^2$.

$ab^2c^2 + 2abc^2 + a^2b^2c^2 = (-2 \times 3 \times 3 \times -1 \times -1)$
$+ (2 \times -2 \times 3 \times -1 \times -1) + (-2 \times -2 \times 3 \times 3 \times -1 \times -1)$
$= -18 - 12 + 36 = 6$.

27. $2a^2bc + 3abc + a^2b^2c^2$.

$2a^2bc + 3abc + a^2b^2c^2 = (2 \times -2 \times -2 \times 3 \times -1) + (3 \times -2 \times 3 \times -1)$
$+ (-2 \times -2 \times 3 \times 3 \times -1 \times -1) = -24 + 18 + 36 = 30$.

28. $6a^2 + 8a^2b^2 - 5a^2bc$.

$6a^2 + 8a^2b^2 - 5a^2bc = (6 \times -2 \times -2) + (8 \times -2 \times -2 \times 3 \times 3)$
$- (5 \times -2 \times -2 \times 3 \times -1) = 24 + 288 + 60 = 372$.

Exercise 17. Page 45.

Divide :

1. x^3 by x.

$$\frac{x^3}{x} = x^2.$$

2. $21x^5$ by $7x^3$.

$$\frac{21x^5}{7x^3} = 3x^2.$$

3. $35x^2$ by $-5x^2$.

$$\frac{35x^2}{-5x^2} = -7.$$

4. $-42x^2$ by $6x^2$.

$$\frac{-42x^2}{6x^2} = -7.$$

5. $-63x^5$ by $-9x$.

$$\frac{-63x^5}{-9x} = 7x^4.$$

6. $-72x^3$ by $-8x^2$.

$$\frac{-72x^3}{-8x^2} = 9x.$$

7. $-32a^2b^2$ by $8ab^2$.

$$\frac{-32a^2b^2}{8ab^2} = -4a.$$

8. $-16x^3y^3$ by $-4xy$.

$$\frac{-16x^3y^3}{-4xy} = 4x^2y^2.$$

9. $18\,x^2y$ by $-2\,xy$.

$$\frac{18\,x^2y}{-2\,xy} = -9\,x.$$

10. $-25\,x^4y^2$ by $5\,x^3y^2$.

$$\frac{-25\,x^4y^2}{-5\,x^3y^2} = 5\,x.$$

11. $-51\,x^2y^3$ by $-17\,x^2y$.

$$\frac{-51\,x^2y^3}{-17\,x^2y} = 3\,y^2.$$

12. $-28\,a^4b^3$ by $7\,a^3b$.

$$\frac{-28\,a^4b^3}{7\,a^3b} = -4\,ab^2.$$

13. $-36\,x^2y^6$ by $-3\,xy^2$.

$$\frac{-36\,x^2y^6}{-3\,xy^2} = 12\,xy^4.$$

14. $-3\,x^4y^6$ by $-5\,xy^3$.

$$\frac{-3\,x^4y^6}{-5\,xy^3} = \frac{3\,x^3y^3}{5}.$$

15. $-12\,a^2b^3$ by $8\,ab^3$.

$$\frac{-12\,a^2b^3}{8\,ab^3} = -\frac{3\,a}{2}.$$

16. $-abcd$ by ac.

$$\frac{-abcd}{ac} = -bd.$$

17. $-a^2b^3c^4d^5$ by $-ab^3c^3d^3$.

$$\frac{-a^2b^3c^4d^5}{-ab^3c^3d^3} = acd^2.$$

18. $2\,x^2y^2z^3$ by $-3\,xyz^3$.

$$\frac{2\,x^2y^2z^3}{-3\,xyz^3} = -\frac{2\,xy}{3}.$$

19. $-5\,a^5b^3c^7$ by $-a^4b^2c^7$.

$$\frac{-5\,a^5b^3c^7}{-a^4b^2c^7} = 5\,ab.$$

20. $52\,a^2m^3n^4$ by $13\,a^2m^2n^3$.

$$\frac{52\,a^2m^3n^4}{13\,a^2m^2n^3} = 4\,mn.$$

21. $13xy^2z^4$ by $39\,xyz$.

$$\frac{13\,xy^2z^4}{39\,xyz} = \frac{yz^3}{3}.$$

22. $68\,xc^2d^3$ by $-4\,xcd^2$.

$$\frac{68\,xc^2d^3}{-4\,xcd^2} = -17\,cd.$$

23. $-8\,m^5n^3p^2$ by $-4\,m^5np$.

$$\frac{-8\,m^5n^3p^2}{-4\,m^5np} = 2\,n^2p.$$

24. $-6\,pqr^3$ by $-2\,p^2qr$.

$$\frac{-6\,pqr^3}{-2\,p^2qr} = \frac{3\,r^2}{p}.$$

25. $26\,a^2g^2t^5$ by $-2\,agt^4$.

$$\frac{26\,a^2g^2t^5}{-2\,agt^4} = -13\,agt.$$

26. $-a^4b^2c^3$ by $-a^5b^3c^4$.

$$\frac{-a^4b^2c^3}{-a^5b^3c^4} = \frac{1}{abc}.$$

27. $-3\,x^2y^2z^2$ by $-2\,x^3y^4z^5$.

$$\frac{-3\,x^2y^2z^2}{-2\,x^3y^4z^5} = \frac{3}{2\,xy^2z^3}.$$

28. $-6\,mnp$ by $-3\,m^2n^2p^2$.

$$\frac{-6\,mnp}{-3\,m^2n^2p^2} = \frac{2}{mnp}.$$

29. $-17\,a^2b^3c^4$ by $51\,ab^5c^4$.

$$\frac{-17\,a^2b^3c^4}{51\,ab^5c^4} = -\frac{a}{3\,b^2}.$$

30. $-19\,mg^2t^3$ by $57m^2gt^4$.

$$\frac{-19\,mg^2t^3}{57\,m^2gt^4} = -\frac{g}{3\,mt}.$$

Exercise 18. Page 48.

Find the sum of:

1. $a^2 - ab + b^2$; $a^2 + ab + b^2$.

$$\begin{array}{r}
a^2 - ab + b^2 \\
a^2 + ab + b^2 \\
\hline
2a^2 \qquad + 2b^2
\end{array}$$

2. $3a^2 + 5a - 7$; $6a^2 - 7a + 13$.

$$\begin{array}{r}
3a^2 + 5a - 7 \\
6a^2 - 7a + 13 \\
\hline
9a^2 - 2a + 6
\end{array}$$

3. $x + 2y - 3z$; $-3x + y + 2z$; $2x - 3y + z$.

$$\begin{array}{r}
x + 2y - 3z \\
-3x + y + 2z \\
2x - 3y + z \\
\hline
0
\end{array}$$

4. $3x + 2y - z$; $-x + 3y + 2z$; $2x - y + 3z$.

$$\begin{array}{r}
3x + 2y - z \\
-x + 3y + 2z \\
2x - y + 3z \\
\hline
4x + 4y + 4z
\end{array}$$

5. $-3a + 2b + c$; $a - 3b + 2c$; $2a + 3b - c$.

$$\begin{array}{r}
-3a + 2b + c \\
a - 3b + 2c \\
2a + 3b - c \\
\hline
2b + 2c
\end{array}$$

6. $-a + 3b + 4c$; $3a - b + 2c$; $2a + 2b - 2c$.

$$\begin{array}{r}
-a + 3b + 4c \\
3a - b + 2c \\
2a + 2b - 2c \\
\hline
4a + 4b + 4c
\end{array}$$

7. $4a^2 + 3a + 5$; $-2a^2 + 3a - 8$; $a^2 - a + 1$.

$$\begin{array}{r}
4a^2 + 3a + 5 \\
-2a^2 + 3a - 8 \\
a^2 - a + 1 \\
\hline
3a^2 + 5a - 2
\end{array}$$

8. $5ab + 6bc - 7ac$; $3ab - 9bc + 4ac$; $3bc + 6ac$.

$$\begin{array}{r}
5ab + 6bc - 7ac \\
3ab - 9bc + 4ac \\
3bc + 6ac \\
\hline
8ab \qquad + 3ac
\end{array}$$

9. $x^3 + x^2 + x$; $2x^3 + 3x^2 - 2x$; $3x^3 - 4x^2 + x$.

$$\begin{array}{l}
x^3 + x^2 + x \\
2x^3 + 3x^2 - 2x \\
3x^3 - 4x^2 + x \\
\hline
6x^3
\end{array}$$

10. $3y^2 - x^2 - 3xy$; $5x^2 + 6xy - 7y^2$; $x^2 + 2y^2$.

$$\begin{array}{r}
-\ x^2 - 3xy + 3y^2 \\
5x^2 + 6xy - 7y^2 \\
x^2 \qquad + 2y^2 \\
\hline
5x^2 + 3xy - 2y^2
\end{array}$$

11. $2a^2 - 2ab + 3b^2$; $4b^2 + 5ab - 2a^2$; $a^2 - 3ab - 9b^2$.

$$\begin{array}{r}
2a^2 - 2ab + 3b^2 \\
-2a^2 + 5ab + 4b^2 \\
a^2 - 3ab - 9b^2 \\
\hline
a^2 \qquad - 2b^2
\end{array}$$

12. $a^3 - a^2 + a - 1$; $a^2 - 2a + 2$; $3a^3 + 7a + 1$.

$$\begin{array}{r}
a^3 - a^2 + a - 1 \\
a^2 - 2a + 2 \\
3a^3 \qquad + 7a + 1 \\
\hline
4a^3 \qquad + 6a + 2
\end{array}$$

13. $2m^3 - m^2 - m$; $4m^3 + 8m^2 - 7$; $-3m^3 + m + 9$.

$$\begin{array}{r}
2m^3 - m^2 - m \qquad \\
4m^3 + 8m^2 \qquad - 7 \\
-3m^3 \qquad + m + 9 \\
\hline
3m^3 + 7m^2 \qquad + 2
\end{array}$$

14. $x^3 - 3x + 6y$; $x^2 + 2x - 5y$; $x^3 - 3x^2 + 5x$.

$$\begin{array}{r}
x^3 \qquad - 3x + 6y \\
x^2 + 2x - 5y \\
x^3 - 3x^2 + 5x \qquad \\
\hline
2x^3 - 2x^2 + 4x + y
\end{array}$$

15. $6x^3 - 5x + 1$; $x^3 + 3x + 4$; $7x^2 + 2x - 3$.

$$\begin{array}{r}
6x^3 \qquad\quad -5x+1 \\
x^3 \qquad\quad +3x+4 \\
7x^2+2x-3 \\
\hline
7x^3+7x^2 \qquad\quad +2
\end{array}$$

16. $a^3 + 3a^2b - 3ab^2$; $-3a^2b - 6ab^2 - b^3$; $3a^2b + 4ab^2$.

$$\begin{array}{r}
a^3+3a^2b-3ab^2 \qquad \\
-3a^2b-6ab^2-b^3 \\
3a^2b+4ab^2 \qquad \\
\hline
a^3+3a^2b-5ab^2-b^3
\end{array}$$

17. $a^3 - 2a^2b - 2ab^2$; $a^2b - 3ab^2 - b^3$; $3ab^2 - 2a^3 - b^3$.

$$\begin{array}{rrrr}
a^3 & -2a^2b & -2ab^2 & \\
 & a^2b & -3ab^2 & -b^3 \\
-2a^3 & & +3ab^2 & -b^3 \\
\hline
-a^3 & -a^2b & -2ab^2 & -2b^3
\end{array}$$

18. $7x^3 - 2x^2y + 9xy^2 + 13y^3$; $5x^2y - 4xy^2 - 2x^3 - 3y^3$; $y^3 - x^3 - 3x^2y - 5xy^2$; $2x^2y - 5y^3 - 2x^3 - xy^2$.

$$\begin{array}{rrrr}
7x^3 & -2x^2y & +9xy^2 & +13y^3 \\
-2x^3 & +5x^2y & -4xy^2 & -3y^3 \\
-x^3 & -3x^2y & -5xy^2 & +y^3 \\
-2x^3 & +2x^2y & -xy^2 & -5y^3 \\
\hline
2x^3 & +2x^2y & -xy^2 & +6y^3
\end{array}$$

19. Show that $x + y + z = 0$, if $x = a - b - c$, $y = 2b + 2c - 3a$, and $z = 2a - b - c$.

$$\begin{array}{rrrr}
x = & a & -b & -c \\
y = & -3a & +2b & +2c \\
z = & 2a & -b & -c \\
\hline
\therefore x+y+z = & & 0 &
\end{array}$$

20. Show that $x + y = 3z$, if $x = 3a^2 - 6a + 12$, $y = 9a^2 + 12a - 21$, and $z = 4a^2 + 2a - 3$.

$$\begin{array}{rrrr}
x = & 3a^2 & -6a & +12 \\
y = & 9a^2 & +12a & -21 \\
\hline
\therefore x+y = & 12a^2 & +6a & -9
\end{array}$$

$$\begin{array}{rrrr}
z = & 4a^2 & +2a & -3 \\
z = & 4a^2 & +2a & -3 \\
z = & 4a^2 & +2a & -3 \\
\hline
3z = & 12a^2 & +6a & -9
\end{array}$$

$$\therefore x + y = 3z.$$

Exercise 19. Page 50.

Subtract:

1. $a - 2b + 3c$ from $2a - 3b + 4c$.

$$\begin{array}{r}2a - 3b + 4c\\ a - 2b + 3c\\ \hline a - b + c\end{array}$$

2. $a - 3b - 5c$ from $3a - 5b + c$.

$$\begin{array}{r}3a - 5b + c\\ a - 3b - 5c\\ \hline 2a - 2b + 6c\end{array}$$

3. $2x - 4y + 6z$ from $4x - y - 2z$.

$$\begin{array}{r}4x - y - 2z\\ 2x - 4y + 6z\\ \hline 2x + 3y - 8z\end{array}$$

4. $5x - 11y - 3z$ from $6x - 7y + 2z$.

$$\begin{array}{r}6x - 7y + 2z\\ 5x - 11y - 3z\\ \hline x + 4y + 5z\end{array}$$

5. $ab - ac - bc + bd$ from $ab + ac + bc + bd$.

$$\begin{array}{r}ab + ac + bc + bd\\ ab - ac - bc + bd\\ \hline 2ac + 2bc\end{array}$$

6. $3ab + 2ac - 3bc + bd$ from $5ab - ac + bc + bd$.

$$\begin{array}{r}5ab - ac + bc + bd\\ 3ab + 2ac - 3bc + bd\\ \hline 2ab - 3ac + 4bc\end{array}$$

7. $2x^3 - x^2 - 5x + 3$ from $3x^3 + 2x^2 - 3x - 5$.

$$\begin{array}{r}3x^3 + 2x^2 - 3x - 5\\ 2x^3 - x^2 - 5x + 3\\ \hline x^3 + 3x^2 + 2x - 8\end{array}$$

8. $7x^2 - 5x + 1 - a$ from $x^3 - x + 1 - a$.

$$\begin{array}{r}x^3 - x + 1 - a\\ 7x^2 - 5x + 1 - a\\ \hline x^3 - 7x^2 + 4x\end{array}$$

9. $7b^3 + 8c^3 - 15abc$ from $9b^3 + 3abc - 7c^3$.

$$\begin{array}{r}
9b^3 + 3abc - 7c^3 \\
7b^3 - 15abc + 8c^3 \\
\hline
2b^3 + 18abc - 15c^3
\end{array}$$

10. $x^4 + x - 5x^3 + 5$ from $7 - 2x^2 - 3x^3 + x^4$.

$$\begin{array}{l}
7 \qquad - 2x^2 - 3x^3 + x^4 \\
5 + x \qquad\quad - 5x^3 + x^4 \\
\hline
2 - x - 2x^2 + 2x^3
\end{array}$$

11. $a^3 + b^3 + c^3 - 3abc$ from $3abc + a^3 - 2b^3 - 3c^3$.

$$\begin{array}{r}
a^3 - 2b^3 - 3c^3 + 3abc \\
a^3 + b^3 + c^3 - 3abc \\
\hline
-3b^3 - 4c^3 + 6abc
\end{array}$$

12. $2x^4 - 5x^2 + 7x - 3$ from $x^4 + 2 - 2x^3 - x^2$.

$$\begin{array}{r}
x^4 - 2x^3 - x^2 \qquad + 2 \\
2x^4 \qquad - 5x^2 + 7x - 3 \\
\hline
- x^4 - 2x^3 + 4x^2 - 7x + 5
\end{array}$$

13. $1 - x^5 - x + x^4 - x^3$ from $x^4 + 1 + x + x^2$.

$$\begin{array}{l}
1 + x + x^2 \qquad + x^4 \\
1 - x \qquad - x^3 + x^4 - x^5 \\
\hline
\quad 2x + x^2 + x^3 \qquad + x^5
\end{array}$$

14. $a^3 - b^3 + 3a^2b - 3ab^2$ from $a^3 + b^3 - a^2b - ab^2$.

$$\begin{array}{r}
a^3 + b^3 - a^2b - ab^2 \\
a^3 - b^3 + 3a^2b - 3ab^2 \\
\hline
2b^3 - 4a^2b + 2ab^2
\end{array}$$

15. $a^2b - ab^2 - 3a^3b^3 - b^4$ from $b^4 - 5a^3b^3 - 2ab^2 + a^2b$.

$$\begin{array}{r}
b^4 - 5a^3b^3 - 2ab^2 + a^2b \\
- b^4 - 3a^3b^3 - ab^2 + a^2b \\
\hline
2b^4 - 2a^3b^3 - ab^2 \qquad
\end{array}$$

16. $-x^3 + 7x^2y - 2y^3 + 3xy^2$ from $3x^3 + 5y^3 - xy^2 + 4x^2y$.

$$\begin{array}{r}
3x^3 + 4x^2y - xy^2 + 5y^3 \\
- x^3 + 7x^2y + 3xy^2 - 2y^3 \\
\hline
4x^3 - 3x^2y - 4xy^2 + 7y^3
\end{array}$$

Exercise 20. Page 52.

... the brackets and collect the like terms:

$- b - c) - a + 2b.$

$- b - c) - a + 2b \qquad = a - b - b + c - a + 2b$

$= a - a - b - b + 2b + c = c.$

$x - (a - b) + a - y].$

$x - (a - b) + a - y] \qquad = x - [x - a + b + a - y]$

$= x - x + a - b - a + y = y - b.$

$2y - [-7c - 2x] + y\}.$

$2y - [-7c - 2x] + y\} \qquad = 3x - \{2y + 7c + 2x + y\}$

$= 3x - 2y - 7c - 2x - y = x - 3y - 7c.$

$- (2b + 5) - 2a].$

$- (2b + 5) - 2a] \qquad = 5a - [7 - 2b - 5 - 2a]$

$= 5a - 7 + 2b + 5 + 2a = 7a + 2b - 2.$

$3a - 2x) - 5a].$

$3a - 2x) - 5a] \qquad = x - [2x + 3a - 2x - 5a]$

$x - 2x - 3a + 2x + 5a = 2a + x.$

$13z + 12x)].$

$(13z + 12x)] \qquad = x - [15y - 13z - 12x]$

$= x - 15y + 13z + 12x = 13x - 15y + 13z.$

$(b + 2c)].$

$(b + 2c)] \qquad = 2a - b + [4c - b - 2c]$

$= 2a - b + 4c - b - 2c = 2a - 2b + 2c.$

$(2b - c)]\}.$

$(2b - c)]\} \qquad = 5a - \{b + [3c - 2b + c]\}$

$(b + 3c - 2b + c\} = 5a - b - 3c + 2b - c$

$= 5a + b - 4c.$

$z]\}.$

$\} \qquad = 7x - \{5y - [3z - 3x - z]\}$

$3x + z\}$

$3x - z = 4x - 5y + 2z.$

10. $(a-b+c)-$ [illegible]

$(a-b+c)-$ [illegible]

$=a-b+c-$ [illegible]

$=3a-b.$

11. $3x-[-2y-(2y-3z$ [illegible]

$3x-[-2y-(2y-3z$ [illegible]

$=3x-[-2y-2y$ [illegible]

$=3x+2y+2y-$ [illegible]

$=2x+3y+z.$

12. $x-[2x+(x-2y)+2y]-3x-$ [illegible]

$x-[2x+(x-2y)+2y]-3x-$ [illegible]

$=x-[2x+x-2y+2y]-$ [illegible]

$=x-2x-x+2y-2y-$ [illegible]

$=-8x+y.$

13. $x-[y+z-x-(x+y)-z]+$ [illegible]

$x-[y+z-x-(x+y)-z]+$ [illegible]

$=x-[y+z-x-x-y-z]+$ [illegible]

$=x-y-z+x+x+y+z-$ [illegible]

$=6x-2y-z.$

Consider *all the factors* that [illegible] x, y, and z as coefficients, and collect them in brackets:

14. $ax+by+cz-ay+az-bz.$

$ax+by+cz-ay+az-bz=$ [illegible]

15. $ax+az+by-cz-ay+cx.$

$ax+az+by-cz-ay+cx=$ [illegible]

16. $2ax-3ay-4by+5cx-6bz-7cz.$

$2ax-3ay-4by+5cx-6bz-7cz$

$=(2a+5c)x-(3a+4b)y-(6b+7c)z.$

17. $az-bmy+3cz-anx-cny+acx.$

$az-bmy+3cz-anx-cny+acx$

$=(ac-an)x-(bm+cn)y+(a+3c)z.$

18. $mnx-x-mny-y+mnz+z.$

$mnx-x-mny-y+mnz+z$

$=(mn-1)x-(mn+1)y+(mn+1)z.$

Exercise 21. Page 53.

First determine the *sign*, then the product of the numerical coefficients, and then the exponent of each letter.

Find the product of:

1. $x + 7$ and x.

$$\begin{array}{r} x + 7 \\ x \\ \hline x^2 + 7x \end{array}$$

2. $2x - 3y$ and $4x$.

$$\begin{array}{r} 2x - 3y \\ 4x \\ \hline 8x^2 - 12xy \end{array}$$

3. $2x - 3y$ and $7y$.

$$\begin{array}{r} 2x - 3y \\ 7y \\ \hline 14xy - 21y^2 \end{array}$$

4. $x - 2a$ and $2a$.

$$\begin{array}{r} x - 2a \\ 2a \\ \hline 2ax - 4a^2 \end{array}$$

5. $-x + 3b$ and $-b$.

$$\begin{array}{r} -x + 3b \\ -b \\ \hline bx - 3b^2 \end{array}$$

6. $2a^2 - 3ab$ and $-3a$.

$$\begin{array}{r} 2a^2 - 3ab \\ -3a \\ \hline -6a^3 + 9a^2b \end{array}$$

7. $2x^2 + 3xz$ and $5z$.

$$\begin{array}{r} 2x^2 + 3xz \\ 5z \\ \hline 10x^2z + 15xz^2 \end{array}$$

8. $a^2 - 5ab$ and $5ab$.

$$\begin{array}{r} a^2 - 5ab \\ 5ab \\ \hline 5a^3b - 25a^2b^2 \end{array}$$

9. $x^2 - 3xy$ and $-y^2$.

$$\begin{array}{r} x^2 - 3xy \\ -y^2 \\ \hline -x^2y^2 + 3xy^3 \end{array}$$

10. $2x^3 - 3x^2$ and $2x^2$.

$$\begin{array}{r} 2x^3 - 3x^2 \\ 2x^2 \\ \hline 4x^5 - 6x^4 \end{array}$$

11. $x^2 - 3y^2$ and $4y$.

$$\begin{array}{r} x^2 - 3y^2 \\ 4y \\ \hline 4x^2y - 12y^3 \end{array}$$

12. $x^2 - 3y^2$ and $-x^2$.

$$\begin{array}{r} x^2 - 3y^2 \\ -x^2 \\ \hline -x^4 + 3x^2y^2 \end{array}$$

13. $b^3 - a^2b^2$ and $-a^3$.

$$\begin{array}{r} b^3 - a^2b^2 \\ -a^3 \\ \hline -a^3b^3 + a^5b^2 \end{array}$$

14. $-a^2b^2 - a^3$ and $-a^2$.

$$\begin{array}{r} -a^2b^2 - a^3 \\ -a^2 \\ \hline a^4b^2 + a^5 \end{array}$$

15. $2x^3 - 3x^2 + x$ and $2x^2$.

$$\begin{array}{r} 2x^3 - 3x^2 + x \\ 2x^2 \\ \hline 4x^5 - 6x^4 + 2x^3 \end{array}$$

16. $a^2 - 5ab - b^2$ and $5ab$.

$$\begin{array}{r} a^2 - 5ab - b^2 \\ 5ab \\ \hline 5a^3b - 25a^2b^2 - 5ab^3 \end{array}$$

17. $a^3 + 2a^2b + 2ab^2$ and a^2.

$$\begin{array}{r} a^3 + 2a^2b + 2ab^2 \\ a^2 \\ \hline a^5 + 2a^4b + 2a^3b^2 \end{array}$$

18. $a^3 + 2a^2b + 2ab^2$ and b^3.

$$\begin{array}{r} a^3 + 2a^2b + 2ab^2 \\ b^3 \\ \hline a^3b^3 + 2a^2b^4 + 2ab^5 \end{array}$$

19. $4x^2 - 6xy - 9y^2$ and $2x$.

$$\begin{array}{r} 4x^2 - 6xy - 9y^2 \\ 2x \\ \hline 8x^3 - 12x^2y - 18xy^2 \end{array}$$

20. $-x^2 - 2xy + y^2$ and $-y$.

$$\begin{array}{r} -x^2 - 2xy + y^2 \\ -y \\ \hline x^2y + 2xy^2 - y^3 \end{array}$$

21. $-a^3 - a^2b^2 - b^3$ and $-a^2$.

$$\begin{array}{r} -a^3 - a^2b^2 - b^3 \\ -a^2 \\ \hline a^5 + a^4b^2 + a^2b^3 \end{array}$$

22. $-x^2 + 2xy - y^2$ and $-y^2$.

$$\begin{array}{r} -x^2 + 2xy - y^2 \\ -y^2 \\ \hline x^2y^2 - 2xy^3 + y^4 \end{array}$$

23. $3a^2b^2 - 4ab^3 + a^3b$ and $5a^2b^2$.

$$\begin{array}{r} 3a^2b^2 - 4ab^3 + a^3b \\ 5a^2b^2 \\ \hline 15a^4b^4 - 20a^3b^5 + 5a^5b^3 \end{array}$$

24. $-ax^2 + 3axy^2 - ay^4$ and $-3ay^2$.

$$\begin{array}{r} -ax^2 + 3axy^2 - ay^4 \\ -3ay^2 \\ \hline 3a^2x^2y^2 - 9a^2xy^4 + 3a^2y^6 \end{array}$$

25. $x^{12} - x^{10}y^3 - x^3y^{10}$ and x^3y^2.

$$\begin{array}{r} x^{12} - x^{10}y^3 - x^3y^{10} \\ x^3y^2 \\ \hline x^{15}y^2 - x^{13}y^5 - x^6y^{12} \end{array}$$

26. $-2x^3 + 3x^2y^2 - 2xy^3$ and $-2x^2y^3$.

$$\begin{array}{r} -2x^3 + 3x^2y^2 - 2xy^3 \\ -2x^2y^3 \\ \hline 4x^5y^3 - 6x^4y^5 + 4x^3y^6 \end{array}$$

27. $a^3x^2y^5 - a^2xy^4 - ay^3$ and $a^7x^3y^5$.

$$\begin{array}{r} a^3x^2y^5 - a^2xy^4 - ay^3 \\ a^7x^3y^5 \\ \hline a^{10}x^5y^{10} - a^9x^4y^9 - a^8x^3y^8 \end{array}$$

28. $3a^2b^2 - 2ab^3 + 5a^3b$ and $5a^2b^3$.

$$\begin{array}{r} 3a^2b^2 - 2ab^3 + 5a^3b \\ 5a^2b^3 \\ \hline 15a^4b^5 - 10a^3b^6 + 25a^5b^4 \end{array}$$

Exercise 22. Page 56.

Find the product of:

1. $x + 7$ and $x + 6$.

$$\begin{array}{rrr} x & +7 & \\ x & +6 & \\ \hline x^2 & +7x & \\ & 6x & +42 \\ \hline x^2 & +13x & +42 \end{array}$$

2. $x - 7$ and $x + 6$.

$$\begin{array}{rrr} x & -7 & \\ x & +6 & \\ \hline x^2 & -7x & \\ & 6x & -42 \\ \hline x^2 & -x & -42 \end{array}$$

3. $x + 7$ and $x - 6$.

$$\begin{array}{rrr} x & +7 & \\ x & -6 & \\ \hline x^2 & +7x & \\ & -6x & -42 \\ \hline x^2 & +x & -42 \end{array}$$

4. $x - 7$ and $x - 6$.

$$\begin{array}{rrr} x & -7 & \\ x & -6 & \\ \hline x^2 & -7x & \\ & -6x & +42 \\ \hline x^2 & -13x & +42 \end{array}$$

5. $x + 8$ and $x - 5$.

$$\begin{array}{rrr} x & +8 & \\ x & -5 & \\ \hline x^2 & +8x & \\ & -5x & -40 \\ \hline x^2 & +3x & -40 \end{array}$$

6. $2x + 3$ and $2x + 3$.

$$\begin{array}{rrr} 2x & +3 & \\ 2x & +3 & \\ \hline 4x^2 & +6x & \\ & 6x & +9 \\ \hline 4x^2 & +12x & +9 \end{array}$$

7. $2x - 3$ and $2x - 3$.

$$\begin{array}{rrr} 2x & -3 & \\ 2x & -3 & \\ \hline 4x^2 & -6x & \\ & -6x & +9 \\ \hline 4x^2 & -12x & +9 \end{array}$$

8. $2x + 3$ and $2x - 3$.

$$\begin{array}{rrr} 2x & +3 & \\ 2x & -3 & \\ \hline 4x^2 & +6x & \\ & -6x & -9 \\ \hline 4x^2 & & -9 \end{array}$$

9. $3x - 2$ and $2 - 3x$.

$$\begin{array}{rrr} 3x & -2 & \\ -3x & +2 & \\ \hline -9x^2 & +6x & \\ & 6x & -4 \\ \hline -9x^2 & +12x & -4 \end{array}$$

10. $5x - 3$ and $4x - 7$.

$$\begin{array}{rrr} 5x & -3 & \\ 4x & -7 & \\ \hline 20x^2 & -12x & \\ & -35x & +21 \\ \hline 20x^2 & -47x & +21 \end{array}$$

11. $a-2b$ and $a+3b$.

$$\begin{array}{rrr}
a^{} & -2b & \\
a^{} & +3b & \\ \hline
a^2 & -2ab & \\
 & 3ab & -6b^2 \\ \hline
a^2 & +\ ab & -6b^2
\end{array}$$

12. $a-7b$ and $a-5b$.

$$\begin{array}{rrr}
a^{} & -\ 7b & \\
a^{} & -\ 5b & \\ \hline
a^2 & -\ 7ab & \\
 & -\ 5ab & +35b^2 \\ \hline
a^2 & -12ab & +35b^2
\end{array}$$

13. $5x-3y$ and $5x-3y$.

$$\begin{array}{rrr}
5x & -\ 3y & \\
5x & -\ 3y & \\ \hline
25x^2 & -15xy & \\
 & -15xy & +9y^2 \\ \hline
25x^2 & -30xy & +9y^2
\end{array}$$

14. $x-b$ and $x-c$.

$$\begin{array}{rrrr}
x & -b & & \\
x & -c & & \\ \hline
x^2 & -bx & & \\
 & & -cx & +bc \\ \hline
x^2 & -bx & -cx & +bc
\end{array}$$

15. $2m-p$ and $4m-3p$.

$$\begin{array}{rrr}
2m & -\ p & \\
4m & -\ 3p & \\ \hline
8m^2 & -\ 4mp & \\
 & -\ 6mp & +3p^2 \\ \hline
8m^2 & -10mp & +3p^2
\end{array}$$

16. $a+b+c$ and $a-c$.

$$\begin{array}{rrrrr}
a & +b & +c & & \\
a & & -c & & \\ \hline
a^2 & +ab & +ac & & \\
 & & -ac & -bc & -c^2 \\ \hline
a^2 & +ab & & -bc & -c^2
\end{array}$$

17. a^2-ab+b^2 and a^2+b^2.

$$\begin{array}{rrrrr}
a^2 & -ab & +b^2 & & \\
 & a^2 & +b^2 & & \\ \hline
a^4 & -a^3b & +a^2b^2 & & \\
 & & a^2b^2 & -ab^3 & +b^4 \\ \hline
a^4 & -a^3b & +2a^2b^2 & -ab^3 & +b^4
\end{array}$$

18. x^3-3x^2+7 and x^2-3.

$$\begin{array}{rrrrr}
 & x^3 & -3x^2 & +7 & \\
 & & x^2 & -3 & \\ \hline
x^5 & -3x^4 & +7x^2 & & \\
 & -3x^3 & +9x^2 & -21 & \\ \hline
x^5 & -3x^4 & -3x^3 & +16x^2 & -21
\end{array}$$

19. a^2+ab+b^2 and $a-b$.

$$\begin{array}{rrrr}
a^2 & +ab & +b^2 & \\
 & a & -b & \\ \hline
a^3 & +a^2b & +ab^2 & \\
 & -a^2b & -ab^2 & -b^3 \\ \hline
a^3 & & & -b^3
\end{array}$$

20. a^2-ab+b^2 and $a+b$.

$$\begin{array}{rrrr}
a^2 & -ab & +b^2 & \\
 & a & +b & \\ \hline
a^3 & -a^2b & +ab^2 & \\
 & a^2b & -ab^2 & +b^3 \\ \hline
a^3 & & & +b^3
\end{array}$$

21. $x^2+5x-10$ and $2x^2+3x-4$.

$$\begin{array}{rrrrr}
 & & x^2 & +5x & -10 \\
 & & 2x^2 & +3x & -4 \\ \hline
2x^4 & +10x^3 & -20x^2 & & \\
 & 3x^3 & +15x^2 & -30x & \\
 & & -4x^2 & -20x & +40 \\ \hline
2x^4 & +13x^3 & -9x^2 & -50x & +40
\end{array}$$

22. $3x^3-2x^2+x$ and $3x^2+2x-2$.

$$\begin{array}{rrrrr}
3x^3 & -2x^2 & +x & & \\
3x^2 & +2x & -2 & & \\ \hline
9x^5 & -6x^4 & +3x^3 & & \\
 & 6x^4 & -4x^3 & +2x^2 & \\
 & & -6x^3 & +4x^2 & -2x \\ \hline
9x^5 & & -7x^3 & +6x^2 & -2x
\end{array}$$

23. $x^3+2x^2y+3xy^2$ and $x^2-2xy+y^2$.

$$\begin{array}{rrrrr}
x^3 & +2x^2y & +3xy^2 & & \\
x^2 & -2xy & +y^2 & & \\ \hline
x^5 & +2x^4y & +3x^3y^2 & & \\
 & -2x^4y & -4x^3y^2 & -6x^2y^3 & \\
 & & x^3y^2 & +2x^2y^3 & +3xy^4 \\ \hline
x^5 & & & -4x^2y^3 & +3xy^4
\end{array}$$

24. $a^2-3ab-b^2$ and $-a^2+ab+2b^2$.

$$\begin{array}{rrrrr}
 & a^2 & -3ab & -b^2 & \\
 & -a^2 & +ab & +2b^2 & \\ \hline
-a^4 & +3a^3b & +a^2b^2 & & \\
 & a^3b & -3a^2b^2 & -ab^3 & \\
 & & 2a^2b^2 & -6ab^3 & -2b^4 \\ \hline
-a^4 & +4a^3b & & -7ab^3 & -2b^4
\end{array}$$

25. $3a^2b^2+2ab^3-5a^3b$ and $5a^2b^2-ab^3-b^4$.

$$\begin{array}{rrrrr}
-5a^3b & +3a^2b^2 & +2ab^3 & & \\
5a^2b^2 & -ab^3 & -b^4 & & \\ \hline
-25a^5b^3 & +15a^4b^4 & +10a^3b^5 & & \\
 & 5a^4b^4 & -3a^3b^5 & -2a^2b^6 & \\
 & & 5a^3b^5 & -3a^2b^6 & -2ab^7 \\ \hline
-25a^5b^3 & +20a^4b^4 & +12a^3b^5 & -5a^2b^6 & -2ab^7
\end{array}$$

26. $a^2 - 2ab + b^2$ and $a^2 + 2ab + b^2$.

$$
\begin{array}{lllll}
a^2 & -2ab & +b^2 & & \\
a^2 & +2ab & +b^2 & & \\
\hline
a^4 & -2a^3b & +a^2b^2 & & \\
 & 2a^3b & -4a^2b^2 & +2ab^3 & \\
 & & a^2b^2 & -2ab^3 & +b^4 \\
\hline
a^4 & & -2a^2b^2 & & +b^4
\end{array}
$$

27. $ab + ac + cd$ and $ab - ac + cd$.

$$
\begin{array}{llllll}
ab & +ac & +cd & & & \\
ab & -ac & +cd & & & \\
\hline
a^2b^2 & +a^2bc & +abcd & & & \\
 & -a^2bc & & -a^2c^2 & -ac^2d & \\
 & & abcd & & +ac^2d & +c^2d^2 \\
\hline
a^2b^2 & & +2abcd & -a^2c^2 & & +c^2d^2
\end{array}
$$

28. $3x^2y^2 + xy^3 - 2x^3y$ and $x^2y^2 + xy^3 - 3y^4$.

$$
\begin{array}{lllll}
-2x^3y & +3x^2y^2 & +xy^3 & & \\
 x^2y^2 & +xy^3 & -3y^4 & & \\
\hline
-2x^5y^3 & +3x^4y^4 & +x^3y^5 & & \\
 & -2x^4y^4 & +3x^3y^5 & +x^2y^6 & \\
 & & 6x^3y^5 & -9x^2y^6 & -3xy^7 \\
\hline
-2x^5y^3 & +x^4y^4 & +10x^3y^5 & -8x^2y^6 & -3xy^7
\end{array}
$$

29. $x^2 + 2xy - y^2$ and $x^2 - 2xy + y^2$.

$$
\begin{array}{lllll}
x^2 & +2xy & -y^2 & & \\
x^2 & -2xy & +y^2 & & \\
\hline
x^4 & +2x^3y & -x^2y^2 & & \\
 & -2x^3y & -4x^2y^2 & +2xy^3 & \\
 & & +x^2y^2 & +2xy^3 & -y^4 \\
\hline
x^4 & & -4x^2y^2 & +4xy^3 & -y^4
\end{array}
$$

30. $3x^2 + xy - y^2$ and $x^2 - 2xy - 3y^2$.

$$
\begin{array}{lllll}
3x^2 & +xy & -y^2 & & \\
x^2 & -2xy & -3y^2 & & \\
\hline
3x^4 & +x^3y & -x^2y^2 & & \\
 & -6x^3y & -2x^2y^2 & +2xy^3 & \\
 & & -9x^2y^2 & -3xy^3 & +3y^4 \\
\hline
3x^4 & -5x^3y & -12x^2y^2 & -xy^3 & +3y^4
\end{array}
$$

Exercise 20. Page 52.

Remove the brackets and collect the like terms:

1. $a-b-(b-c)-a+2b.$

$a-b-(b-c)-a+2b \qquad = a-b-b+c-a+2b$
$= a-a-b-b+2b+c=c.$

2. $x-[x-(a-b)+a-y].$

$x-[x-(a-b)+a-y] \qquad = x-[x-a+b+a-y]$
$= x-x+a-b-a+y=y-b.$

3. $3x-\{2y-[-7c-2x]+y\}.$

$3x-\{2y-[-7c-2x]+y\} \qquad = 3x-\{2y+7c+2x+y\}$
$= 3x-2y-7c-2x-y=x-3y-7c.$

4. $5a-[7-(2b+5)-2a].$

$5a-[7-(2b+5)-2a] \qquad = 5a-[7-2b-5-2a]$
$= 5a-7+2b+5+2a=7a+2b-2.$

5. $x-[2x+(3a-2x)-5a].$

$x-[2x+(3a-2x)-5a] \qquad = x-[2x+3a-2x-5a]$
$= x-2x-3a+2x+5a=2a+x.$

6. $x-[15y-(13z+12x)].$

$x-[15y-(13x+12x)] \qquad = x-[15y-13z-12x]$
$= x-15y+13z+12x=13x-15y+13z.$

7. $2a-b+[4c-(b+2c)].$

$2a-b+[4c-(b+2c)] \qquad = 2a-b+[4c-b-2c]$
$= 2a-b+4c-b-2c=2a-2b+2c.$

8. $5a-\{b+[3c-(2b-c)]\}.$

$5a-\{b+[3c-(2b-c)]\} \qquad = 5a-\{b+[3c-2b+c]\}$
$= 5a-\{b+3c-2b+c\}=5a-b-3c+2b-c$
$= 5a+b-4c.$

9. $7x-\{5y-[3z-(3x+z)]\}.$

$7x-\{5y-[3z-(3x+z)]\} \qquad = 7x-\{5y-[3z-3x-z]\}$
$= 7x-\{5y-3z+3x+z\}$
$= 7x-5y+3z-3x-z=4x-5y+2z.$

10. $(a-b+c)-(b-a-c)+(a+b-2c)$.

$$\begin{aligned}&(a-b+c)-(b-a-c)+(a+b-2c)\\&\quad=a-b+c-b+a+c+a+b-2c\\&\quad=3a-b.\end{aligned}$$

11. $3x-[-2y-(2y-3x)+z]+[x-(y-2z-x)]$.

$$\begin{aligned}&3x-[-2y-(2y-3x)+z]+[x-(y-2z-x)]\\&\quad=3x-[-2y-2y+3x+z]+[x-y+2z+x]\\&\quad=3x+2y+2y-3x-z+x-y+2z+x\\&\quad=2x+3y+z.\end{aligned}$$

12. $x-[2x+(x-2y)+2y]-3x-\{4x-[(x+2y)-y]\}$.

$$\begin{aligned}&x-[2x+(x-2y)+2y]-3x-\{4x-[(x+2y)-y]\}\\&\quad=x-[2x+x-2y+2y]-3x-\{4x-x-2y+y\}\\&\quad=x-2x-x+2y-2y-3x-4x+x+2y-y\\&\quad=-8x+y.\end{aligned}$$

13. $x-[y+z-x-(x+y)-z]+(3x-\overline{2y+z})$.

$$\begin{aligned}&x-[y+z-x-(x+y)-z]+(3x-\overline{2y+z})\\&\quad=x-[y+z-x-x-y-z]+(3x-2y-z)\\&\quad=x-y-z+x+x+y+z+3x-2y-z\\&\quad=6x-2y-z.\end{aligned}$$

Consider *all the factors* that precede x, y, and z as *coefficients*, and collect them in brackets:

14. $ax+by+cz-ay+az-bx$.

$$ax+by+cz-ay+az-bx=(a-b)x-(a-b)y+(a+c)z.$$

15. $ax+az+by-cz-ay+cx$.

$$ax+az+by-cz-ay+cx=(a+c)x-(a-b)y+(a-c)z.$$

16. $2ax-3ay-4by+5cx-6bz-7cz$.

$$\begin{aligned}&2ax-3ay-4by+5cx-6bz-7cz\\&\quad=(2a+5c)x-(3a+4b)y-(6b+7c)z.\end{aligned}$$

17. $az-bmy+3cz-anx-cny+acx$.

$$\begin{aligned}&az-bmy+3cz-anx-cny+acx\\&\quad=(ac-an)x-(bm+cn)y+(a+3c)z.\end{aligned}$$

18. $mnx-x-mny-y+mnz+z$.

$$\begin{aligned}&mnx-x-mny-y+mnz+z\\&\quad=(mn-1)x-(mn+1)y+(mn+1)z.\end{aligned}$$

Exercise 21. Page 53.

First determine the *sign*, then the product of the numerical coefficients, and then the exponent of each letter.

Find the product of:

1. $x + 7$ and x.

$$\begin{array}{r} x + 7 \\ x \\ \hline x^2 + 7x \end{array}$$

2. $2x - 3y$ and $4x$.

$$\begin{array}{r} 2x - 3y \\ 4x \\ \hline 8x^2 - 12xy \end{array}$$

3. $2x - 3y$ and $7y$.

$$\begin{array}{r} 2x - 3y \\ 7y \\ \hline 14xy - 21y^2 \end{array}$$

4. $x - 2a$ and $2a$.

$$\begin{array}{r} x - 2a \\ 2a \\ \hline 2ax - 4a^2 \end{array}$$

5. $-x + 3b$ and $-b$.

$$\begin{array}{r} -x + 3b \\ -b \\ \hline bx - 3b^2 \end{array}$$

6. $2a^2 - 3ab$ and $-3a$.

$$\begin{array}{r} 2a^2 - 3ab \\ -3a \\ \hline -6a^3 + 9a^2b \end{array}$$

7. $2x^2 + 3xz$ and $5z$.

$$\begin{array}{r} 2x^2 + 3xz \\ 5z \\ \hline 10x^2z + 15xz^2 \end{array}$$

8. $a^2 - 5ab$ and $5ab$.

$$\begin{array}{r} a^2 - 5ab \\ 5ab \\ \hline 5a^3b - 25a^2b^2 \end{array}$$

9. $x^2 - 3xy$ and $-y^2$.

$$\begin{array}{r} x^2 - 3xy \\ -y^2 \\ \hline -x^2y^2 + 3xy^3 \end{array}$$

10. $2x^3 - 3x^2$ and $2x^2$.

$$\begin{array}{r} 2x^3 - 3x^2 \\ 2x^2 \\ \hline 4x^5 - 6x^4 \end{array}$$

11. $x^2 - 3y^2$ and $4y$.

$$\begin{array}{r} x^2 - 3y^2 \\ 4y \\ \hline 4x^2y - 12y^3 \end{array}$$

12. $x^2 - 3y^2$ and $-x^2$.

$$\begin{array}{r} x^2 - 3y^2 \\ -x^2 \\ \hline -x^4 + 3x^2y^2 \end{array}$$

13. $b^3 - a^2b^2$ and $-a^3$.

$$\begin{array}{r} b^3 - a^2b^2 \\ -a^3 \\ \hline -a^3b^3 + a^5b^2 \end{array}$$

14. $-a^2b^2 - a^3$ and $-a^2$.

$$\begin{array}{r} -a^2b^2 - a^3 \\ -a^2 \\ \hline a^4b^2 + a^5 \end{array}$$

15. $2x^3 - 3x^2 + x$ and $2x^2$.

$$\begin{array}{r} 2x^3 - 3x^2 + x \\ 2x^2 \\ \hline 4x^5 - 6x^4 + 2x^3 \end{array}$$

16. $a^2 - 5ab - b^2$ and $5ab$.

$$\begin{array}{r} a^2 - 5ab - b^2 \\ 5ab \\ \hline 5a^3b - 25a^2b^2 - 5ab^3 \end{array}$$

17. $a^3 + 2a^2b + 2ab^2$ and a^2.

$$\begin{array}{r} a^3 + 2a^2b + 2ab^2 \\ a^2 \\ \hline a^5 + 2a^4b + 2a^3b^2 \end{array}$$

18. $a^3 + 2a^2b + 2ab^2$ and b^3.

$$\begin{array}{r} a^3 + 2a^2b + 2ab^2 \\ b^3 \\ \hline a^3b^3 + 2a^2b^4 + 2ab^5 \end{array}$$

19. $4x^2 - 6xy - 9y^2$ and $2x$.

$$\begin{array}{r} 4x^2 - 6xy - 9y^2 \\ 2x \\ \hline 8x^3 - 12x^2y - 18xy^2 \end{array}$$

20. $-x^2 - 2xy + y^2$ and $-y$.

$$\begin{array}{r} -x^2 - 2xy + y^2 \\ -y \\ \hline x^2y + 2xy^2 - y^3 \end{array}$$

21. $-a^3 - a^2b^2 - b^3$ and $-a^2$.

$$\begin{array}{r} -a^3 - a^2b^2 - b^3 \\ -a^2 \\ \hline a^5 + a^4b^2 + a^2b^3 \end{array}$$

22. $-x^2 + 2xy - y^2$ and $-y^2$.

$$\begin{array}{r} -x^2 + 2xy - y^2 \\ -y^2 \\ \hline x^2y^2 - 2xy^3 + y^4 \end{array}$$

23. $3a^2b^2 - 4ab^3 + a^3b$ and $5a^2b^2$.

$$\begin{array}{r} 3a^2b^2 - 4ab^3 + a^3b \\ 5a^2b^2 \\ \hline 15a^4b^4 - 20a^3b^5 + 5a^5b^3 \end{array}$$

24. $-ax^2 + 3axy^2 - ay^4$ and $-3ay^2$.

$$\begin{array}{r} -ax^2 + 3axy^2 - ay^4 \\ -3ay^2 \\ \hline 3a^2x^2y^2 - 9a^2xy^4 + 3a^2y^6 \end{array}$$

25. $x^{12} - x^{10}y^3 - x^3y^{10}$ and x^3y^2.

$$\begin{array}{r} x^{12} - x^{10}y^3 - x^3y^{10} \\ x^3y^2 \\ \hline x^{15}y^2 - x^{13}y^5 - x^6y^{12} \end{array}$$

26. $-2x^3 + 3x^2y^2 - 2xy^3$ and $-2x^2y^3$.

$$\begin{array}{r} -2x^3 + 3x^2y^2 - 2xy^3 \\ -2x^2y^3 \\ \hline 4x^5y^3 - 6x^4y^5 + 4x^3y^6 \end{array}$$

27. $a^3x^2y^5 - a^2xy^4 - ay^3$ and $a^7x^3y^5$.

$$\begin{array}{r} a^3x^2y^5 - a^2xy^4 - ay^3 \\ a^7x^3y^5 \\ \hline a^{10}x^5y^{10} - a^9x^4y^9 - a^8x^3y^8 \end{array}$$

28. $3a^2b^2 - 2ab^3 + 5a^3b$ and $5a^2b^3$.

$$\begin{array}{r} 3a^2b^2 - 2ab^3 + 5a^3b \\ 5a^2b^3 \\ \hline 15a^4b^5 - 10a^3b^6 + 25a^5b^4 \end{array}$$

12. x^3-8x-3 by $x-3$.

$$\begin{array}{r|l}
x^3 \qquad\quad -8x-3 & x-3 \\ \cline{2-2}
\underline{x^3-3x^2 \qquad\qquad\;} & x^2+3x+1 \\
3x^2-8x-3 & \\
\underline{3x^2-9x \quad\;\;} & \\
x-3 & \\
\underline{x-3} &
\end{array}$$

13. $a^2+2ab+b^2-c^2$ by $a-b-c$.

$$\begin{array}{r|l}
a^2-2ab \qquad +b^2-c^2 & a-b-c \\ \cline{2-2}
\underline{a^2-\;\; ab-ac \qquad\quad} & a-b+c \\
-\;\; ab+ac+b^2-c^2 & \\
\underline{-\;\; ab+bc+b^2 \quad\;\;} & \\
ac-bc-c^2 & \\
\underline{ac-bc-c^2} &
\end{array}$$

14. $a^2+2ab+b^2-c^2$ by $a+b+c$.

$$\begin{array}{r|l}
a^2+2ab \qquad +b^2-c^2 & a+b+c \\ \cline{2-2}
\underline{a^2+\;\; ab+ac \qquad\quad} & a+b-c \\
ab-ac+b^2-c^2 & \\
\underline{ab+bc+b^2 \quad\;\;} & \\
-ac-bc-c^2 & \\
\underline{-ac-bc-c^2} &
\end{array}$$

15. $x^2-y^2+2yz-z^2$ by $x-y+z$.

$$\begin{array}{r|l}
x^2 \qquad\qquad -y^2+2yz-z^2 & x-y+z \\ \cline{2-2}
\underline{x^2-xy+xz \qquad\qquad\qquad\;} & x+y-z \\
xy-xz-y^2+2yz-z^2 & \\
\underline{xy \qquad -y^2+\;\; yz \quad\;\;} & \\
-xz \qquad +\;\; yz-z^2 & \\
\underline{-xz \qquad +\;\; yz-z^2} &
\end{array}$$

16. c^4+2c^2-c+2 by c^2-c+1.

$$\begin{array}{r|l}
c^4 \qquad +2c^2-\;\; c+2 & c^2-c+1 \\ \cline{2-2}
\underline{c^4-c^3+\;\; c^2 \qquad\quad\;} & c^2+c+2 \\
c^3+\;\; c^2-\;\; c+2 & \\
\underline{c^3-\;\; c^2+\;\; c \quad\;} & \\
2c^2-2c+2 & \\
\underline{2c^2-2c+2} &
\end{array}$$

17. $x^2 - 4y^2 - 4yz - z^2$ by $x + 2y + z$.

$$\begin{array}{l|l}
x^2 \qquad\qquad - 4y^2 - 4yz - z^2 & x + 2y + z \\
\underline{x^2 + 2xy + xz} & \overline{x - 2y - z} \\
\quad - 2xy - xz - 4y^2 - 4yz - z^2 & \\
\quad \underline{- 2xy \qquad - 4y^2 - 2yz} & \\
\qquad\quad - xz \qquad - 2yz - z^2 & \\
\qquad\quad \underline{- xz \qquad - 2yz - z^2} &
\end{array}$$

Arrange and divide:

18. $x^3 - 6a^3 + 11a^2x - 6ax^2$ by $x^2 + 6a^2 - 5ax$.

$$\begin{array}{l|l}
x^3 - 6ax^2 + 11a^2x - 6a^3 & x^2 - 5ax + 6a^2 \\
\underline{x^3 - 5ax^2 + \ \ 6a^2x} & \overline{x \ - a} \\
\quad - \ \ ax^2 + \ \ 5a^2x - 6a^3 & \\
\quad \underline{- \ \ ax^2 + \ \ 5a^2x - 6a^3} &
\end{array}$$

19. $a^2 - 4b^2 - 9c^2 + 12bc$ by $a - 3c + 2b$.

$$\begin{array}{l|l}
a^2 \qquad\qquad - 4b^2 + 12bc - 9c^2 & a + 2b - 3c \\
\underline{a^2 + 2ab - 3ac} & \overline{a - 2b + 3c} \\
\quad - 2ab + 3ac - 4b^2 + 12bc - 9c^2 & \\
\quad \underline{- 2ab \qquad - 4b^2 + \ \ 6bc} & \\
\qquad\qquad 3ac \qquad + \ \ 6bc - 9c^2 & \\
\qquad\qquad \underline{3ac \qquad + \ \ 6bc - 9c^2} &
\end{array}$$

20. $2a^3 - 8a + a^4 + 12 - 7a^2$ by $2 + a^2 - 3a$.

$$\begin{array}{l|l}
a^4 + 2a^3 - \ \ 7a^2 - \ \ 8a + 12 & a^2 - 3a + 2 \\
\underline{a^4 - 3a^3 + \ \ 2a^2} & \overline{a^2 + 5a + 6} \\
\qquad 5a^3 - \ \ 9a^2 - \ \ 8a + 12 & \\
\qquad \underline{5a^3 - 15a^2 + 10a} & \\
\qquad\qquad 6a^2 - 18a + 12 & \\
\qquad\qquad \underline{6a^2 - 18a + 12} &
\end{array}$$

21. $q^4 + 6q^3 + 4 + 12q + 13q^2$ by $3q + 2 + q^2$.

$$\begin{array}{l|l}
q^4 + 6q^3 + 13q^2 + 12q + 4 & q^2 + 3q + 2 \\
\underline{q^4 + 3q^3 + \ \ 2q^2} & \overline{q^2 + 3q + 2} \\
\qquad 3q^3 + 11q^2 + 12q + 4 & \\
\qquad \underline{3q^3 + \ \ 9q^2 + \ \ 6q} & \\
\qquad\qquad 2q^2 + \ \ 6q + 4 & \\
\qquad\qquad \underline{2q^2 + \ \ 6q + 4} &
\end{array}$$

22. $27a^3 - 8b^3$ by $3a - 2b$.

$$
\begin{array}{l|l}
27a^3 \qquad\qquad\qquad\quad - 8b^3 & 3a - 2b \\ \cline{2-2}
\underline{27a^3 - 18a^2b} & 9a^2 + 6ab + 4b^2 \\
\qquad\quad 18a^2b \qquad\quad - 8b^3 & \\
\qquad\quad \underline{18a^2b - 12ab^2} & \\
\qquad\qquad\qquad 12ab^2 - 8b^3 & \\
\qquad\qquad\qquad \underline{12ab^2 - 8b^3} &
\end{array}
$$

Find the remainder when:

23. $a^4 + 9a^2 + 15 - 11a - 7a^3$ is divided by $a - 5$.

$$
\begin{array}{l|l}
a^4 - 7a^3 + 9a^2 - 11a + 15 & a - 5 \\ \cline{2-2}
\underline{a^4 - 5a^3} & a^3 - 2a^2 - a - 16 \\
\quad - 2a^3 + 9a^2 - 11a + 15 & \\
\quad \underline{- 2a^3 + 10a^2} & \\
\qquad\quad - a^2 - 11a + 15 & \\
\qquad\quad \underline{- a^2 + 5a} & \\
\qquad\qquad\quad - 16a + 15 & \\
\qquad\qquad\quad \underline{- 16a + 80} & \\
\qquad\qquad\qquad\quad - 65 &
\end{array}
$$

24. $7 - 8c^2 + 5c^3 + 8c$ is divided by $5c - 3$.

$$
\begin{array}{l|l}
5c^3 - 8c^2 + 8c + 7 & 5c - 3 \\ \cline{2-2}
\underline{5c^3 - 3c^2} & c^2 - c + 1 \\
\quad - 5c^2 + 8c + 7 & \\
\quad \underline{- 5c^2 + 3c} & \\
\qquad\qquad 5c + 7 & \\
\qquad\qquad \underline{5c - 3} & \\
\qquad\qquad\quad 10 &
\end{array}
$$

25. $3 + 11a^3 + 30a^4 - 82a^2 - 5a$ is divided by $3a^2 - 4 + 2a$.

$$
\begin{array}{l|l}
30a^4 + 11a^3 - 82a^2 - 5a + 3 & 3a^2 + 2a - 4 \\ \cline{2-2}
\underline{30a^4 + 20a^3 - 40a^2} & 10a^2 - 3a - 12 \\
\quad - 9a^3 - 42a^2 - 5a + 3 & \\
\quad \underline{- 9a^3 - 6a^2 + 12a} & \\
\qquad\quad - 36a^2 - 17a + 3 & \\
\qquad\quad \underline{- 36a^2 - 24a + 48} & \\
\qquad\qquad\qquad 7a - 45 &
\end{array}
$$

26. $2x^3 - 16x + 10 - 39x^2 + 17x^4$ is divided by $2 - 5x^2 - 4x$.

$$
\begin{array}{rrrrr|l}
10 & -16x & -39x^2 & +\ 2x^3 & +17x^4 & 2 - 4x - 5x^2 \\
10 & -20x & -25x^2 & & & \overline{5 + 2x - 3x^2} \\
\hline
 & 4x & -14x^2 & +\ 2x^3 & +17x^4 & \\
 & 4x & -\ 8x^2 & -10x^3 & & \\
\hline
 & & -\ 6x^2 & +12x^3 & +17x^4 & \\
 & & -\ 6x^2 & +12x^3 & +15x^4 & \\
\hline
 & & & & 2x^4 &
\end{array}
$$

Exercise 25. Page 63.

Miscellaneous Examples.

1. Add $2a^2 - 3ac - 3ab$; $2b^2 + 3ac + a^2$; $-a^2 - 2b^2 + 3ab$.

$$
\begin{array}{rrrr}
2a^2 & -3ac & -3ab & \\
a^2 & +3ac & & +2b^2 \\
-a^2 & & +3ab & -2b^2 \\
\hline
2a^2 & & &
\end{array}
$$

2. Subtract $3a^4 - 2a^3b + 4a^2b^2$ from $4b^4 - 2ab^3 + 4a^2b^2$.

$$
\begin{array}{rrrrr}
 & & 4a^2b^2 & -2ab^3 & +4b^4 \\
3a^4 & -2a^3b & +4a^2b^2 & & \\
\hline
-3a^4 & +2a^3b & & -2ab^3 & +4b^4
\end{array}
$$

3. Simplify $x - y - \{z - x - (y - x + z)\}$.

$$
\begin{aligned}
& x - y - \{z - x - (y - x + z)\} \\
&= x - y - \{z - x - y + x - z\} \\
&= x - y - z + x + y - x + z \\
&= x.
\end{aligned}
$$

4. Multiply $a^2 + b^2 + c^2 - d^2$ by $a^2 + b^2 - c^2 + d^2$.

$$
\begin{array}{rrrrrrrrr}
a^2 & +\ b^2 & +\ c^2 & -\ d^2 & & & & & \\
a^2 & +\ b^2 & -\ c^2 & +\ d^2 & & & & & \\
\hline
a^4 & +a^2b^2 & +a^2c^2 & -a^2d^2 & & & & & \\
 & a^2b^2 & & & +b^4 & +b^2c^2 & -b^2d^2 & & \\
 & & -a^2c^2 & & & -b^2c^2 & & -c^4 & +c^2d^2 \\
 & & & a^2d^2 & & & +b^2d^2 & & +\ c^2d^2 - d^4 \\
\hline
a^4 & +2a^2b^2 & & & +b^4 & & & -c^4 & +2c^2d^2 - d^4
\end{array}
$$

5. Divide $10\,y^6 + 2 - 12\,y^5$ by $1 + y^2 - 2\,y$.

$$
\begin{array}{l|l}
10y^6-12y^5 \qquad\qquad\qquad\qquad +2 & y^2-2y+1 \\
\underline{10y^6-20y^5+10y^4} & \overline{10y^4+8y^3+6y^2+4y+2} \\
\quad 8y^5-10y^4 \qquad\qquad\quad +2 & \\
\quad \underline{8y^5-16y^4+\ 8y^3} & \\
\qquad 6y^4-\ 8y^3 \qquad\quad +2 & \\
\qquad \underline{6y^4-12y^3+6y^2} & \\
\qquad\quad 4y^3-6y^2 \quad\ +2 & \\
\qquad\quad \underline{4y^3-8y^2+4y} & \\
\qquad\qquad 2y^2-4y+2 & \\
\qquad\qquad \underline{2y^2-4y+2} &
\end{array}
$$

6. If $a = 1$, $b = 2$, and $c = -3$, find the value of $a^2 + b^2 + c^2 + 2\,ab + 2\,ac + 2\,bc$.

$$a^2 + b^2 + c^2 + 2\,ab + 2\,ac + 2\,bc = 1 + 4 + 9 + 4 - 6 - 12 = 0.$$

7. Simplify $x - (y - z) - \{4\,y + [2\,y - (z - x)]\}$.

$$
\begin{aligned}
& x - (y - z) - \{4\,y + [2\,y - (z - x)]\} \\
& = x - y + z - \{4\,y + [2\,y - z + x]\} \\
& = x - y + z - \{4\,y + 2\,y - z + x\} \\
& = x - y + z - 4\,y - 2\,y + z - x \\
& = 2\,z - 7\,y.
\end{aligned}
$$

8. Multiply $a^2 + b^2 + c^2 - ab - ac - bc$ by $a + b + c$.

$$
\begin{array}{llllllllll}
a^2 & -ab & -ac & +\ b^2 & -\ bc & +c^2 & & & & \\
\underline{a} & \underline{+\ b} & \underline{+\ c} & & & & & & & \\
a^3 & -a^2b & -a^2c & +ab^2 & -\ abc & +ac^2 & & & & \\
 & a^2b & & -ab^2 & -\ abc & & +b^3 & -b^2c & +bc^2 & \\
 & & a^2c & & -\ abc & -ac^2 & & +b^2c & -bc^2 & +c^3 \\
\hline
a^3 & & & & -3\,abc & & +b^3 & & & +c^3
\end{array}
$$

9. Divide $16\,y^4 - 21\,x^2y^2 + 21\,x^3y - 10\,x^4$ by $4\,y^2 - 5\,x^2 + 3\,xy$.

$$
\begin{array}{l|l}
16\,y^4 \qquad\qquad\quad -21\,x^2y^2 + 21\,x^3y - 10\,x^4 & 4\,y^2 + 3\,xy - 5\,x^2 \\
\underline{16\,y^4 + 12\,xy^3 - 20\,x^2y^2} & \overline{4\,y^2 - 3\,xy + 2\,x^2} \\
\quad -12\,xy^3 - \quad x^2y^2 + 21\,x^3y - 10\,x^4 & \\
\quad \underline{-12\,xy^3 - \ 9\,x^2y^2 + 15\,x^3y} & \\
\qquad\qquad\quad 8\,x^2y^2 + \ 6\,x^2y - 10\,x^4 & \\
\qquad\qquad\quad \underline{8\,x^2y^2 + \ 6\,x^3y - 10\,x^4} &
\end{array}
$$

10. Add $-2a^4+3a^3b-4a^2b^2$; $2a^3b-3a^2b^2$; $7a^2b^2+2a^4-b^4$.

$$\begin{array}{rrrr}
-2a^4 & +3a^3b & -4a^2b^2 & \\
 & 2a^3b & -3a^2b^2 & \\
2a^4 & & +7a^2b^2 & -b^4 \\
\hline
 & 5a^3b & & -b^4
\end{array}$$

11. From $3x^3+5x-1$ take the sum of $x-5+5x^2$ and $3+4x-3x^2$.

$$\begin{array}{lrrr}
 & 5x^2 & +x & -5 \\
 & -3x^2 & +4x & +3 \\
\hline
\text{Sum} = & 2x^2 & +5x & -2
\end{array}
\qquad
\begin{array}{lrrrr}
 & 3x^3 & & +5x & -1 \\
 & & 2x^2 & +5x & -2 \\
\hline
\text{Dif.} = & 3x^3 & -2x^2 & & +1
\end{array}$$

12. The minuend is $9c^2+11c-5$, and the remainder is $6c^2-13c+7$. What is the subtrahend?

$$\begin{array}{rrr}
9c^2 & +11c & -5 \\
6c^2 & -13c & +7 \\
\hline
3c^2 & +24c & -12
\end{array}$$

13. Find the remainder when a^4+6b^4 is divided by $a^2+2ab+2b^2$.

$$\begin{array}{rrrrr|l}
a^4 & & & & +6b^4 & a^2+2ab+2b^2 \\
\cline{6-6}
a^4 & +2a^3b & +2a^2b^2 & & & a^2-2ab+2b^2 \\
\cline{1-5}
 & -2a^3b & -2a^2b^2 & & +6b^4 & \\
 & -2a^3b & -4a^2b^2 & -4ab^3 & & \\
\cline{2-5}
 & & 2a^2b^2 & +4ab^3 & +6b^4 & \\
 & & 2a^2b^2 & +4ab^3 & +6b^4 & \\
\cline{3-5}
 & & & & 2b^4 &
\end{array}$$

14. Multiply $2-5x^2-4x$ by $5+2x-3x^2$.

$$\begin{array}{rrrrr}
2 & -4x & -5x^2 & & \\
5 & +2x & -3x^2 & & \\
\hline
10 & -20x & -25x^2 & & \\
 & 4x & -8x^2 & -10x^3 & \\
 & & -6x^2 & +12x^3 & +15x^4 \\
\hline
10 & -16x & -39x^2 & +2x^3 & +15x^4
\end{array}$$

15. Divide $a^6+a^5x+a^4x^2-a^3x^3+x^6$ by a^2+ax+x^2.

$$\begin{array}{rrrrrrr|l}
a^6 & +a^5x & +a^4x^2 & -a^3x^3 & & & +x^6 & a^2+ax+x^2 \\
\cline{8-8}
a^6 & +a^5x & +a^4x^2 & & & & & a^4-ax^3+x^4 \\
\cline{1-7}
 & & & -a^3x^3 & & & +x^6 & \\
 & & & -a^3x^3 & -a^2x^4 & -ax^5 & & \\
\cline{4-7}
 & & & & a^2x^4 & +ax^5 & +x^6 & \\
 & & & & a^2x^4 & +ax^5 & +x^6 & \\
\cline{5-7}
\end{array}$$

Bracket the coefficients of the different powers of x:

16. $ax^3 - cx + bx^2 - bx^3 + cx^2 - x.$

$$ax^3 - cx + bx^2 - bx^3 + cx^2 - x$$
$$= (a - b)x^3 + (b + c)x^2 - (c + 1)x.$$

17. $ax^4 - 2x + bx^4 - cx - ax^3 + bx^3.$

$$ax^4 - 2x + bx^4 - cx - ax^3 + bx^3$$
$$= (a + b)x^4 - (a - b)x^3 - (c + 2)x.$$

18. $x^3 - bx^2 - cx + bx - cx^2 + ax^3.$

$$x^3 - bx^2 - cx + bx - cx^2 + ax^3$$
$$= (a + 1)x^3 - (b + c)x^2 + (b - c)x.$$

Exercise 26. Page 65.

Write by inspection the product of:

1. $(m + n)^2.$

$(m + n)^2 = m^2 + 2mn + n^2.$

2. $(c - a)^2.$

$(c - a)^2 = c^2 - 2ac + a^2.$

3. $(a + 2c)^2.$

$(a + 2c)^2 = a^2 + 4ac + 4c^2.$

4. $(3a - 2b)^2.$

$(3a - 2b)^2 = 9a^2 - 12ab + 4b^2.$

5. $(2a + 3b)^2.$

$(2a + 3b)^2 = 4a^2 + 12ab + 9b^2.$

6. $(a - 3b)^2.$

$(a - 3b)^2 = a^2 - 6ab + 9b^2.$

7. $(2x - y)^2.$

$(2x - y)^2 = 4x^2 - 4xy + y^2.$

8. $(y - 2x)^2.$

$(y - 2x)^2 = y^2 - 4xy + 4x^2.$

9. $(a + 5b)^2.$

$(a + 5b)^2 = a^2 + 10ab + 25b^2.$

10. $(2a - 5c)^2$.

$(2a - 5c)^2 = 4a^2 - 20ac + 25c^2$.

11. $(x + y)(x - y)$.

$(x + y)(x - y) = x^2 - y^2$.

12. $(4a - b)(4a + b)$.

$(4a - b)(4a + b) = 16a^2 - b^2$.

13. $(2b - 3c)(2b + 3c)$.

$(2b - 3c)(2b + 3c) = 4b^2 - 9c^2$.

14. $(x + 5b)(x + 5b)$.

$(x + 5b)(x + 5b) = x^2 + 10bx + 25b^2$.

15. $(y - 2z)(y - 2z)$.

$(y - 2z)(y - 2z) = y^2 - 4yz + 4z^2$.

16. $(y + 3z)(y - 3z)$.

$(y + 3z)(y - 3z) = y^2 - 9z^2$.

17. $(2a - 3b)(2a + 3b)$.

$(2a - 3b)(2a + 3b) = 4a^2 - 9b^2$.

18. $(2a - 3b)(2a - 3b)$.

$(2a - 3b)(2a - 3b) = 4a^2 - 12ab + 9b^2$.

19. $(2a + 3b)(2a + 3b)$.

$(2a + 3b)(2a + 3b) = 4a^2 + 12ab + 9b^2$.

20. $(5x + 3a)(5x - 3a)$.

$(5x + 3a)(5x - 3a) = 25x^2 - 9a^2$.

Exercise 27. Page 67.

Write by inspection the product of:

1. $(x + 7)(x + 4)$.

$(x + 7)(x + 4) = x^2 + 11x + 28$.

2. $(x - 3)(x + 7)$.

$(x - 3)(x + 7) = x^2 + 4x - 21$.

3. $(x - 2)(x - 4)$.

$(x - 2)(x - 4) = x^2 - 6x + 8$.

4. $(x-6)(x-10)$.

$(x-6)(x-10) = x^2 - 16x + 60$.

5. $(x+7)(x-4)$.

$(x+7)(x-4) = x^2 + 3x - 28$.

6. $(x+a)(x-2a)$.

$(x+a)(x-2a) = x^2 - ax - 2a^2$.

7. $(x+3a)(x-a)$.

$(x+3a)(x-a) = x^2 + 2ax - 3a^2$.

8. $(a+3c)(a+3c)$.

$(a+3c)(a+3c) = a^2 + 6ac + 9c^2$.

9. $(a+2x)(a-4x)$.

$(a+2x)(a-4x) = a^2 - 2ax - 8x^2$.

10. $(a-3b)(a-4b)$.

$(a-3b)(a-4b) = a^2 - 7ab + 12b^2$.

11. $(a^2-c)(a^2+2c)$.

$(a^2-c)(a^2+2c) = a^4 + a^2c - 2c^2$.

12. $(x-17)(x-3)$.

$(x-17)(x-3) = x^2 - 20x + 51$.

13. $(x+6y)(x-5y)$.

$(x+6y)(x-5y) = x^2 + xy - 30y^2$.

14. $(3+2x)(3-x)$.

$(3+2x)(3-x) = 9 + 3x - 2x^2$.

15. $(5+2x)(1-2x)$.

$(5+2x)(1-2x) = 5 - 8x - 4x^2$.

16. $(a-2b)(a+3b)$.

$(a-2b)(a+3b) = a^2 + ab - 6b^2$.

17. $(a^2b^2-x^2)(a^2b^2-5x^2)$.

$(a^2b^2-x^2)(a^2b^2-5x^2) = a^4b^4 - 6a^2b^2x^2 + 5x^4$.

18. $(a^3b-ab^3)(a^3b+5ab^3)$.

$(a^3b-ab^3)(a^3b+5ab^3) = a^6b^2 + 4a^4b^4 - 5a^2b^6$.

19. $(x^2y - xy^2)(x^2y - 3xy^2)$.
$(x^2y - xy^2)(x^2y - 3xy^2) = x^4y^2 - 4x^3y^3 + 3x^2y^4$.

20. $(x^2y + xy^2)(x^2y + xy^2)$.
$(x^2y + xy^2)(x^2y + xy^2) = x^4y^2 + 2x^3y^3 + x^2y^4$.

21. $(x + a)(x + b)$.
$(x + a)(x + b) = x^2 + (a + b)x + ab$.

22. $(x + a)(x - b)$.
$(x + a)(x - b) = x^2 + (a - b)x - ab$.

23. $(x - a)(x + b)$.
$(x - a)(x + b) = x^2 - (a - b)x - ab$.

24. $(x - a)(x - b)$.
$(x - a)(x - b) = x^2 - (a + b)x + ab$.

25. $(x + 2a)(x + 2b)$.
$(x + 2a)(x + 2b) = x^2 + (2a + 2b)x + 4ab$.

26. $(x - 2a)(x + 2b)$.
$(x - 2a)(x + 2b) = x^2 - (2a - 2b)x - 4ab$.

27. $(x + 2a)(x - 2b)$.
$(x + 2a)(x - 2b) = x^2 + (2a - 2b)x - 4ab$.

28. $(x - 2a)(x - 2b)$.
$(x - 2a)(x - 2b) = x^2 - (2a + 2b)x + 4ab$.

29. $(x - a)(x + 3a)$.
$(x - a)(x + 3a) = x^2 + 2ax - 3a^2$.

30. $(x - 2a)(x + 3a)$.
$(x - 2a)(x + 3a) = x^2 + ax - 6a^2$.

Exercise 28. Page 68.

Write by inspection the quotient of:

1. $\dfrac{x^2 - 4}{x - 2}$.

$\dfrac{x^2 - 4}{x - 2} = x + 2$.

2. $\dfrac{x^2 - 4}{x + 2}$.

$\dfrac{x^2 - 4}{x + 2} = x - 2$.

3. $\frac{a^2-9}{a-3}$.

$$\frac{a^2-9}{a-3}=a+3.$$

4. $\frac{a^2-9}{a+3}$.

$$\frac{a^2-9}{a+3}=a-3.$$

5. $\frac{c^2-25}{c-5}$.

$$\frac{c^2-25}{c-5}=c+5.$$

6. $\frac{c^2-25}{c+5}$.

$$\frac{c^2-25}{c+5}=c-5.$$

7. $\frac{49x^2-y^2}{7x-y}$.

$$\frac{49x^2-y^2}{7x-y}=7x+y.$$

8. $\frac{49x^2-y^2}{7x+y}$.

$$\frac{49x^2-y^2}{7x+y}=7x-y.$$

9. $\frac{9b^2-1}{3b-1}$.

$$\frac{9b^2-1}{3b-1}=3b+1.$$

10. $\frac{9b^2-1}{3b+1}$.

$$\frac{9b^2-1}{3b+1}=3b-1.$$

11. $\frac{16x^4-25a^2}{4x^2-5a}$.

$$\frac{16x^4-25a^2}{4x^2-5a}=4x^2+5a.$$

12. $\frac{16x^4-25a^2}{4x^2+5a}$.

$$\frac{16x^4-25a^2}{4x^2+5a}=4x^2-5a.$$

13. $\frac{9x^2-25y^2}{3x-5y}$.

$$\frac{9x^2-25y^2}{3x-5y}=3x+5y.$$

14. $\frac{a^2-(b-c)^2}{a-(b-c)}$.

$$\frac{a^2-(b-c)^2}{a-(b-c)}=a+(b-c).$$

15. $\frac{a^2-(b-c)^2}{a+(b-c)}$.

$$\frac{a^2-(b-c)^2}{a+(b-c)}=a-(b-c).$$

16. $\frac{a^2-(2b-c)^2}{a-(2b-c)}$.

$$\frac{a^2-(2b-c)^2}{a-(2b-c)}=a+(2b-c).$$

17. $\dfrac{(5a-7b)^2-1}{(5a-7b)-1}$.

$$\frac{(5a-7b)^2-1}{(5a-7b)-1}=(5a-7b)+1.$$

18. $\dfrac{(5a-7b)^2-1}{(5a-7b)+1}$.

$$\frac{(5a-7b)^2-1}{(5a-7b)+1}=(5a-7b)-1.$$

19. $\dfrac{z^2-(x-y)^2}{z-(x-y)}$.

$$\frac{z^2-(x-y)^2}{z-(x-y)}=z+(x-y).$$

20. $\dfrac{z^2-(x-y)^2}{z+(x-y)}$.

$$\frac{z^2-(x-y)^2}{z+(x-y)}=z-(x-y).$$

21. $\dfrac{a^2-(2b-c)^2}{a+(2b-c)}$.

$$\frac{a^2-(2b-c)^2}{a+(2b-c)}=a-(2b-c).$$

22. $\dfrac{(x+3y)^2-z^2}{(x+3y)-z}$.

$$\frac{(x+3y)^2-z^2}{(x+3y)-z}=(x+3y)+z.$$

23. $\dfrac{(x+3y)^2-z^2}{x+3y+z}$.

$$\frac{(x+3y)^2-z^2}{x+3y+z}=x+3y-z.$$

24. $\dfrac{(a+2b)^2-4c^2}{(a+2b)-2c}$.

$$\frac{(a+2b)^2-4c^2}{(a+2b)-2c}=(a+2b)+2c.$$

25. $\dfrac{(a+2b)^2-4c^2}{(a+2b)+2c}$.

$$\frac{(a+2b)^2-4c^2}{(a+2b)+2c}=(a+2b)-2c.$$

26. $\dfrac{1-(3x-2y)^2}{1+(3x-2y)}$.

$$\frac{1-(3x-2y)^2}{1+(3x-2y)}=1-(3x-2y).$$

Exercise 29. Page 69.

Write by inspection the quotient of:

1. $\dfrac{1-x^3}{1-x}$.

$$\frac{1-x^3}{1-x}=1+x+x^2.$$

2. $\dfrac{1-8a^3}{1-2a}$.

$$\frac{1-8a^3}{1-2a}=1+2a+4a^2.$$

3. $\dfrac{1-27c^3}{1-3c}$.

$$\frac{1-27c^3}{1-3c}=1+3c+9c^2.$$

4. $\dfrac{8a^3-b^3}{2a-b}$.

$$\frac{8a^3-b^3}{2a-b}=4a^2+2ab+b^2.$$

5. $\dfrac{64b^3-27c^3}{4b-3c}$.

$$\frac{64b^3-27c^3}{4b-3c}=16b^2+12bc+9c^2.$$

6. $\dfrac{27x^3-8y^3}{3x-2y}$.

$$\frac{27x^3-8y^3}{3x-2y}=9x^2+6xy+4y^2.$$

7. $\dfrac{x^3y^3 - z^3}{xy - z}$.

$$\frac{x^3y^3 - z^3}{xy - z} = x^2y^2 + xyz + z^2.$$

8. $\dfrac{a^3b^3 - 8}{ab - 2}$.

$$\frac{a^3b^3 - 8}{ab - 2} = a^2b^2 + 2\,ab + 4.$$

9. $\dfrac{125\,a^3 - b^3}{5\,a - b}$.

$$\frac{125\,a^3 - b^3}{5\,a - b} = 25\,a^2 + 5\,ab + b^2.$$

10. $\dfrac{a^3 - 8\,b^3}{a - 2\,b}$.

$$\frac{a^3 - 2\,b^3}{a - 2\,b} = a^2 + 2\,ab + 4\,b^2.$$

11. $\dfrac{a^3 - 64}{a - 4}$.

$$\frac{a^3 - 64}{a - 4} = a^2 + 4\,a + 16.$$

12. $\dfrac{a^9 - 27}{a^3 - 3}$.

$$\frac{a^9 - 27}{a^3 - 3} = a^6 + 3\,a^3 + 9.$$

13. $\dfrac{a^{12} - x^6y^6}{a^4 - x^2y^2}$.

$$\frac{a^{12} - x^6y^6}{a^4 - x^2y^2} = a^8 + a^4x^2y^2 + x^4y^4.$$

14. $\dfrac{x^{15} - a^9b^9}{x^5 - a^3b^3}$.

$$\frac{x^{15} - a^9b^9}{x^5 - a^3b^3} = x^{10} + x^5a^3b^3 + a^6b^6.$$

15. $\dfrac{27x^3y^3 - z^{12}}{3xy - z^4}$.

$$\frac{27x^3y^3 - z^{12}}{3xy - z^4} = 9x^2y^2 + 3xyz^4 + z^8.$$

16. $\dfrac{x^3y^3z^3 - 1}{xyz - 1}$.

$$\frac{x^3y^3z^3 - 1}{xyz - 1} = x^2y^2z^2 + xyz + 1.$$

17. $\dfrac{8a^3b^3c^3 - 27}{2abc - 3}$.

$$\frac{8a^3b^3c^3 - 27}{2abc - 3} = 4a^2b^2c^2 + 6abc + 9.$$

18. $\dfrac{1 - 64x^3y^3z^3}{1 - 4xyz}$.

$$\frac{1 - 64x^3y^3z^3}{1 - 4xyz} = 1 + 4xyz + 16x^2y^2z^2.$$

Exercise 30. Page 70.

Write by inspection the quotient of:

1. $\dfrac{1 + x^3}{1 + x}$.

$$\frac{1 + x^3}{1 + x} = 1 - x + x^2.$$

2. $\dfrac{1 + 8a^3}{1 + 2a}$.

$$\frac{1 + 8a^3}{1 + 2a} = 1 - 2a + 4a^2.$$

3. $\dfrac{1 + 27c^3}{1 + 3c}$.

$$\frac{1 + 27c^3}{1 + 3c} = 1 - 3c + 9c^2.$$

4. $\dfrac{8a^3 + b^3}{2a + b}$.

$$\frac{8a^3 + b^3}{2a + b} = 4a^2 - 2ab + b^2.$$

5. $\dfrac{64b^3 + 27c^3}{4b + 3c}$.

$$\frac{64b^3 + 27c^3}{4b + 3c} = 16b^2 - 12bc + 9c^2.$$

6. $\dfrac{27x^3 + 8y^3}{3x + 2y}$.

$$\frac{27x^3 + 8y^3}{3x + 2y} = 9x^2 - 6xy + 4y^2.$$

7. $\dfrac{8x^3 + 125y^3}{2x + 5x}$.

$$\frac{8x^3 + 125y^3}{2x + 5x} = 4x^2 - 10xy + 25y^2.$$

8. $\dfrac{x^3y^3 + z^3}{xy + z}$.

$$\frac{x^3y^3 + z^3}{xy + z} = x^2y^2 - xyz + z^2.$$

9. $\dfrac{a^3b^3 + 8}{ab + 2}$.

$$\frac{a^3b^3 + 8}{ab + 2} = a^2b^2 - 2ab + 4.$$

10. $\dfrac{125a^3 + b^3}{5a + b}$.

$$\frac{125a^3 + b^3}{5a + b} = 25a^2 - 5ab + b^2.$$

11. $\dfrac{a^3 + 8b^3}{a + 2b}$.

$$\frac{a^3 + 8b^3}{a + 2b} = a^2 - 2ab + 4b^2.$$

12. $\dfrac{a^6 + 64}{a^2 + 4}$.

$$\frac{a^6 + 64}{a^2 + 4} = a^4 - 4a^2 + 16.$$

13. $\dfrac{a^9 + 27}{a^3 + 3}$.

$$\frac{a^9 + 27}{a^3 + 3} = a^6 - 3a^3 + 9.$$

14. $\dfrac{8a^6+b^3}{2a^2+b}$.

$$\frac{8a^6+b^3}{2a^2+b}=4a^4-2a^2b+b^2.$$

15. $\dfrac{a^{12}+x^6y^6}{a^4+x^2y^2}$.

$$\frac{a^{12}+x^6y^6}{a^4+x^2y^2}=a^8-a^4x^2y^2+x^4y^4.$$

16. $\dfrac{x^{15}+a^9b^9}{x^5+a^3b^3}$.

$$\frac{x^{15}+a^9b^9}{x^5+a^3b^3}=x^{10}-x^5a^3b^3+a^6b^6.$$

17. $\dfrac{27x^3y^3+z^{12}}{3xy+z^4}$.

$$\frac{27x^3y^3+z^{12}}{3xy+z^4}=9x^2y^2-3xyz^4+z^8.$$

18. $\dfrac{x^3y^3z^3+1}{xyz+1}$.

$$\frac{x^3y^3z^3+1}{xyz+1}=x^2y^2z^2-xyz+1.$$

19. $\dfrac{8a^3b^3c^3+27}{2abc+3}$.

$$\frac{8a^3b^3c^3+27}{2abc+3}=4a^2b^2c^2-6abc+9.$$

20. $\dfrac{1+64x^3y^3z^3}{1+4xyz}$.

$$\frac{1+64x^3y^3z^3}{1+4xyz}=1-4xyz+16x^2y^2z^2.$$

21. $\dfrac{1+27a^6b^3c^3}{1+3a^2bc}$.

$$\frac{1+27a^6b^3c^3}{1+3a^2bc}=1-3a^2bc+9a^4b^2c^2.$$

Find by division the quotient of:

22. $\dfrac{x^4 - y^4}{x - y}$.

$$
\begin{array}{llllll|l}
x^4 & & & & & - y^4 & x - y \\
\cline{7-7}
x^4 & - x^3y & & & & & x^3 + x^2y + xy^2 + y^3 \\
\cline{1-6}
& x^3y & & & & - y^4 & \\
& x^3y & - x^2y^2 & & & & \\
\cline{2-6}
& & x^2y^2 & & & - y^4 & \\
& & x^2y^2 & - xy^3 & & & \\
\cline{3-6}
& & & xy^3 & & - y^4 & \\
& & & xy^3 & & - y^4 & \\
\cline{4-6}
\end{array}
$$

Call the pupils' attention to the fact that if the divisor is $x - y$ the terms of the quotient are all positive, if $x + y$ the terms are alternately positive and negative.

23. $\dfrac{x^4 - y^4}{x + y}$.

$$
\begin{array}{llllll|l}
x^4 & & & & & - y^4 & x + y \\
\cline{7-7}
x^4 & + x^3y & & & & & x^3 - x^2y + xy^2 - y^3 \\
\cline{1-6}
& - x^3y & & & & - y^4 & \\
& - x^3y & - x^2y^2 & & & & \\
\cline{2-6}
& & x^2y^2 & & & - y^4 & \\
& & x^2y^2 & + xy^3 & & & \\
\cline{3-6}
& & & - xy^3 & & - y^4 & \\
& & & - xy^3 & & - y^4 & \\
\cline{4-6}
\end{array}
$$

24. $\dfrac{x^5 - y^5}{x - y}$.

$$
\begin{array}{lllllll|l}
x^5 & & & & & & - y^5 & x-y \\
\cline{8-8}
x^5 & - x^4y & & & & & & x^4+x^3y+x^2y^2+xy^3+y^4 \\
\cline{1-7}
& x^4y & & & & & - y^5 & \\
& x^4y & - x^3y^2 & & & & & \\
\cline{2-7}
& & x^3y^2 & & & & - y^5 & \\
& & x^3y^2 & - x^2y^3 & & & & \\
\cline{3-7}
& & & x^2y^3 & & & - y^5 & \\
& & & x^2y^3 & - xy^4 & & & \\
\cline{4-7}
& & & & xy^4 & & - y^5 & \\
& & & & xy^4 & & - y^5 & \\
\cline{5-7}
\end{array}
$$

25. $\dfrac{x^5+y^5}{x+y}$.

$$
\begin{array}{llllll|l}
x^5 & & & & & +y^5 & x+y \\
\cline{7-7}
x^5+x^4y & & & & & & x^4-x^3y+x^2y^2-xy^3+y^4 \\
\cline{1-6}
 & -x^4y & & & & +y^5 & \\
 & -x^4y-x^3y^2 & & & & & \\
\cline{2-6}
 & & x^3y^2 & & & +y^5 & \\
 & & x^3y^2+x^2y^3 & & & & \\
\cline{3-6}
 & & & -x^2y^3 & & +y^5 & \\
 & & & -x^2y^3-xy^4 & & & \\
\cline{4-6}
 & & & & xy^4 & +y^5 & \\
 & & & & xy^4 & +y^5 & \\
\cline{5-6}
\end{array}
$$

26. $\dfrac{x^6-y^6}{x-y}$.

$$
\begin{array}{lllllll|l}
x^6 & & & & & & -y^6 & x-y \\
\cline{8-8}
x^6-x^5y & & & & & & & x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5 \\
\cline{1-7}
 & x^5y & & & & & -y^6 & \\
 & x^5y-x^4y^2 & & & & & & \\
\cline{2-7}
 & & x^4y^2 & & & & -y^6 & \\
 & & x^4y^2-x^3y^3 & & & & & \\
\cline{3-7}
 & & & x^3y^3 & & & -y^6 & \\
 & & & x^3y^3-x^2y^4 & & & & \\
\cline{4-7}
 & & & & x^2y^4 & & -y^6 & \\
 & & & & x^2y^4-xy^5 & & & \\
\cline{5-7}
 & & & & & xy^5 & -y^6 & \\
 & & & & & xy^5 & -y^6 & \\
\cline{6-7}
\end{array}
$$

27. $\dfrac{x^6-y^6}{x+y}$.

$$
\begin{array}{lllllll|l}
x^6 & & & & & & -y^6 & x+y \\
\cline{8-8}
x^6+x^5y & & & & & & & x^5-x^4y+x^3y^2-x^2y^3+xy^4-y^5 \\
\cline{1-7}
 & -x^5y & & & & & -y^6 & \\
 & -x^5y-x^4y^2 & & & & & & \\
\cline{2-7}
 & & x^4y^2 & & & & -y^6 & \\
 & & x^4y^2+x^3y^3 & & & & & \\
\cline{3-7}
 & & & -x^3y^3 & & & -y^6 & \\
 & & & -x^3y^3-x^2y^4 & & & & \\
\cline{4-7}
 & & & & x^2y^4 & & -y^6 & \\
 & & & & x^2y^4+xy^5 & & & \\
\cline{5-7}
 & & & & & -xy^5 & -y^6 & \\
 & & & & & -xy^5 & -y^6 & \\
\cline{6-7}
\end{array}
$$

Exercise 31. Page 72.

Resolve into two factors:

1. $2x^2 - 4x$.

$$2x^2 - 4x = 2x(x - 2).$$

2. $3a^3 - 6a$.

$$3a^3 - 6a = 3a(a^2 - 2).$$

3. $5a^2b^2 - 10a^3b^3$.

$$5a^2b^2 - 10a^3b^3 = 5a^2b^2(1 - 2ab).$$

4. $3x^2y + 4xy^2$.

$$3x^2y + 4xy^2 = xy(3x + 4y).$$

5. $8a^3b^2 + 4a^2b^3$.

$$8a^3b^2 + 4a^2b^3 = 4a^2b^3(2a + b).$$

6. $3a^4 - 12a^2 - 6a^3$.

$$3a^4 - 12a^2 - 6a^3 = 3a^2(a^2 - 4 - 2a).$$

7. $4x^2 - 8x^4 - 12x^5$.

$$4x^2 - 8x^4 - 12x^5 = 4x^2(1 - 2x^2 - 3x^3).$$

8. $5 - 10x^2y^2 + 15x^2y$.

$$5 - 10x^2y^2 + 15x^2y = 5(1 - 2x^2y^2 + 3x^2y).$$

9. $7a^2 + 14a - 21a^3$.

$$7a^2 + 14a - 21a^3 = 7a(a + 2 - 3a^2).$$

10. $3x^3y^3 - 6x^4y^4 - 9x^2y^2$.

$$3x^3y^3 - 6x^4y^4 - 9x^2y^2 = 3x^2y^2(xy - 2x^2y^2 - 3).$$

Exercise 32. Page 73.

Resolve into factors:

1. $x^3 + x^2 + x + 1$.

$$\begin{aligned} x^3 + x^2 + x + 1 &= x^2(x + 1) + 1(x + 1) \\ &= (x^2 + 1)(x + 1). \end{aligned}$$

2. $x^3 - x^2 + x - 1$.

$$\begin{aligned} x^3 - x^2 + x - 1 &= x^2(x - 1) + 1(x - 1) \\ &= (x^2 + 1)(x - 1). \end{aligned}$$

3. $x^2 + xy + xz + yz.$

$$x^2 + xy + xz + yz = x(x + y) + z(x + y)$$
$$= (x + z)(x + y).$$

4. $ax - bx - ay + by.$

$$ax - bx - ay + by = x(a - b) - y(a - b)$$
$$= (x - y)(a - b).$$

5. $a^2 - ac + ab - bc.$

$$a^2 - ac + ab - bc = a(a - c) + b(a - c)$$
$$= (a + b)(a - c).$$

6. $x^2 - bx + 3x - 3b.$

$$x^2 - bx + 3x - 3b = x(x - b) + 3(x - b)$$
$$= (x + 3)(x - b).$$

7. $2x^3 - x^2 + 4x - 2.$

$$2x^3 - x^2 + 4x - 2 = x^2(2x - 1) + 2(2x - 1)$$
$$= (x^2 + 2)(2x - 1).$$

8. $a^2 - 3a - ab + 3b.$

$$a^2 - 3a - ab + 3b = a(a - 3) - b(a - 3)$$
$$= (a - b)(a - 3).$$

9. $6a^2 + 2ab - 3ac - bc.$

$$6a^2 + 2ab - 3ac - bc = 2a(3a + b) - c(3a + b)$$
$$= (2a - c)(3a + b).$$

10. $abxy + cxy + abc + c^2.$

$$abxy + cxy + abc + c^2 = xy(ab + c) + c(ab + c)$$
$$= (xy + c)(ab + c).$$

11. $ax - ay - bx + cy - cx + by.$

$$ax - ay - bx + cy - cx + by = a(x - y) - b(x - y) - c(x - y)$$
$$= (a - b - c)(x - y).$$

12. $(a - b)^2 - 2c(a - b).$

$$(a - b)^2 - 2c(a - b) = (a - b)(a - b) - 2c(a - b)$$
$$= (a - b - 2c)(a - b).$$

Exercise 33. Page 74.

Resolve into factors:

1. $4 - x^2$.

$$4 - x^2 = (2 + x)(2 - x).$$

2. $9 - x^2$.

$$9 - x^2 = (3 + x)(3 - x).$$

3. $9a^2 - x^2$.

$$9a^2 - x^2 = (3a + x)(3a - x).$$

4. $25 - x^2$.

$$25 - x^2 = (5 + x)(5 - x).$$

5. $25x^2 - a^2$.

$$25x^2 - a^2 = (5x + a)(5x - a).$$

6. $16a^4 - 121$.

$$16a^4 - 121 = (4a^2 + 11)(4a^2 - 11).$$

7. $121a^4 - 16$.

$$121a^4 - 16 = (11a^2 + 4)(11a^2 - 4).$$

8. $4a^2b^2 - c^2d^2$.

$$4a^2b^2 - c^2d^2 = (2ab + cd)(2ab - cd).$$

9. $1 - x^2y^2$.

$$1 - x^2y^2 = (1 + xy)(1 - xy).$$

10. $81x^2y^2 - 1$.

$$81x^2y^2 - 1 = (9xy + 1)(9xy - 1).$$

11. $49a^2b^2 - 4$.

$$49a^2b^2 - 4 = (7ab + 2)(7ab - 2).$$

12. $25a^4b^4 - 9$.

$$25a^4b^4 - 9 = (5a^2b^2 + 3)(5a^2b^2 - 3).$$

13. $9a^8b^6 - 16x^{10}$.

$$9a^8b^6 - 16x^{10} = (3a^4b^3 + 4x^5)(3a^4b^3 - 4x^5).$$

14. $144x^2y^2 - 1$.

$$144x^2y^2 - 1 = (12xy + 1)(12xy - 1).$$

15. $100x^6y^2z^4-1.$

$$100x^6y^2z^4-1=(10x^3yz^2+1)(10x^3yz^2-1).$$

16. $1-121a^4b^8c^{12}.$

$$1-121a^4b^8c^{12}=(1+11a^2b^4c^6)(1-11a^2b^4c^6).$$

17. $25a^2-64x^6y^6.$

$$25a^2-64x^6y^6=(5a+8x^3y^3)(5a-8x^3y^3).$$

18. $16x^{16}-25y^{18}.$

$$16x^{16}-25y^{18}=(4x^8+5y^9)(4x^8-5y^9).$$

Find, by resolving into factors, the value of:

19. $(375)^2-(225)^2.$

$$\begin{aligned}(375)^2-(225)^2&=(375+225)(375-225)\\&=600\times 150\\&=90{,}000.\end{aligned}$$

20. $(579)^2-(559)^2.$

$$\begin{aligned}(579)^2-(559)^2&=(579+559)(579-559)\\&=1138\times 20\\&=22{,}760.\end{aligned}$$

21. $(873)^2-(173)^2.$

$$\begin{aligned}(873)^2-(173)^2&=(873+173)(873-173)\\&=1046\times 700\\&=732{,}200.\end{aligned}$$

22. $(101)^2-(99)^2.$

$$\begin{aligned}(101)^2-(99)^2&=(101+99)(101-99)\\&=200\times 2\\&=400.\end{aligned}$$

23. $(7244)^2-(7242)^2.$

$$\begin{aligned}(7244)^2-(7242)^2&=(7244+7242)(7244-7242)\\&=14{,}486\times 2\\&=28{,}972.\end{aligned}$$

24. $(3781)^2-(219)^2.$

$$\begin{aligned}(3781)^2-(219)^2&=(3781+219)(3781-219)\\&=4000\times 3562\\&=14{,}248{,}000.\end{aligned}$$

Exercise 34. Page 75.

Resolve into factors:

1. $(x+y)^2-z^2$.

$$(x+y)^2-z^2=(x+y+z)(x+y-z).$$

2. $(x-y)^2-z^2$.

$$(x-y)^2-z^2=(x-y+z)(x-y-z).$$

3. $z^2-(x+y)^2$.

$$z^2-(x+y)^2=(z+x+y)(z-x-y).$$

4. $z^2-(x-y)^2$.

$$z^2-(x-y)^2=(z+x-y)(z-x+y).$$

5. $(x+y)^2-4z^2$.

$$(x+y)^2-4z^2=(x+y+2z)(x+y-2z).$$

6. $4z^2-(x-y)^2$.

$$4z^2-(x-y)^2=(2z+x-y)(2z-x+y).$$

7. $(a+2b)^2-c^2$.

$$(a+2b)^2-c^2=(a+2b+c)(a+2b-c).$$

8. $(a-2b)^2-c^2$.

$$(a-2b)^2-c^2=(a-2b+c)(a-2b-c).$$

9. $c^2-(a-2b)^2$.

$$c^2-(a-2b)^2=(c+a-2b)(c-a+2b).$$

10. $(2a+5c)^2-1$.

$$(2a+5c)^2-1=(2a+5c+1)(2a+5c-1).$$

11. $1-(2a-5c)^2$.

$$1-(2a-5c)^2=(1+2a-5c)(1-2a+5c).$$

12. $(a+3b)^2-16c^2$.

$$(a+3b)^2-16c^2=(a+3b+4c)(a+3b-4c).$$

13. $(a-5b)^2-9c^2$.

$$(a-5b)^2-9c^2=(a-5b+3c)(a-5b-3c).$$

14. $16c^2-(a-5b)^2$.

$$16c^2-(a-5b)^2=(4c+a-5b)(4c-a+5b).$$

15. $4a^2 - (x + y)^2$.

$$4a^2 - (x + y)^2 = (2a + x + y)(2a - x - y).$$

16. $b^2 - (a - 2x)^2$.

$$b^2 - (a - 2x)^2 = (b + a - 2x)(b - a + 2x).$$

17. $4z^2 - (x + 3y)^2$.

$$4z^2 - (x + 3y)^2 = (2z + x + 3y)(2z - x - 3y).$$

18. $9 - (3a - 7b)^2$.

$$9 - (3a - 7b)^2 = (3 + 3a - 7b)(3 - 3a + 7b).$$

19. $16a^2 - (2b + 5c)^2$.

$$16a^2 - (2b + 5c)^2 = (4a + 2b + 5c)(4a - 2b - 5c).$$

20. $25c^2 - (3a - 2x)^2$.

$$25c^2 - (3a - 2x)^2 = (5c + 3a - 2x)(5c - 3a + 2x).$$

21. $9a^2 - (3b - 5c)^2$.

$$9a^2 - (3b - 5c)^2 = (3a + 3b - 5c)(3a - 3b + 5c).$$

22. $16y^2 - (a - 3c).^2$

$$16y^2 - (a - 3c)^2 = (4y + a - 3c)(4y - a + 3c).$$

23. $49m^2 - (p + 2q)^2$.

$$49m^2 - (p + 2q)^2 = (7m + p + 2q)(7m - p - 2q).$$

24. $36n^2 - (d - 2c)^2$.

$$36n^2 - (d - 2c)^2 = (6n + d - 2c)(6n - d + 2c).$$

25. $(x + y)^2 - (a + b)^2$.

$$(x + y)^2 - (a + b)^2 = (x + y + a + b)(x + y - a - b).$$

26. $(x - y)^2 - (a - b)^2$.

$$(x - y)^2 - (a - b)^2 = (x - y + a - b)(x - y - a + b).$$

27. $(2x + 3)^2 - (2a + b)^2$.

$$(2x + 3)^2 - (2a + b)^2 = (2x + 3 + 2a + b)(2x + 3 - 2a - b).$$

28. $(b - c)^2 - (a - 2x)^2$.

$$(b - c)^2 - (a - 2x)^2 = (b - c + a - 2x)(b - c - a + 2x).$$

29. $(3x - y)^2 - (2a - b)^2$.

$$(3x - y)^2 - (2a - b)^2 = (3x - y + 2a - b)(3x - y - 2a + b).$$

30. $(x-3y)^2-(a+2b)^2$.

$$(x-3y)^2-(a+2b)^2=(x-3y+a+2b)(x-3y-a-2b).$$

31. $(x+2y)^2-(a+3b)^2$.

$$(x+2y)^2-(a+3b)^2=(x+2y+a+3b)(x+2y-a-3b).$$

32. $(x+y)^2-(a-z)^2$.

$$(x+y)^2-(a-z)^2=(x+y+a-z)(x+y-a+z).$$

Exercise 35. Page 77.

Resolve into factors:

1. $8x^3-y^3$.

$$8x^3-y^3=(2x-y)(4x^2+2xy+y^2).$$

2. x^3-1.

$$x^3-1=(x-1)(x^2+x+1).$$

3. $x^3y^3-z^3$.

$$x^3y^3-z^3=(xy-z)(x^2y^2+xyz+z^2).$$

4. x^3-64.

$$x^3-64=(x-4)(x^2+4x+16).$$

5. $125a^3-b^3$.

$$125a^3-b^3=(5a-b)(25a^2+5ab+b^2).$$

6. a^3-343.

$$a^3-343=(a-7)(a^2+7a+49).$$

7. $a^3b^3-27c^3$.

$$a^3b^3-27c^3=(ab-3c)(a^2b^2+3abc+9c^2).$$

8. $x^3y^3z^3-8$.

$$x^3y^3z^3-8=(xyz-2)(x^2y^2z^2+2xyz+4).$$

9. $8a^3b^3-27y^6$.

$$8a^3b^3-27y^6=(2ab-3y^2)(4a^2b^2+6aby^2+9y^4).$$

10. $64x^3-y^9$.

$$64x^3-y^9=(4x-y^3)(16x^2+4xy^3+y^6).$$

11. $27a^3-64c^6$.

$$27a^3-64c^6=(3a-4c^2)(9a^2+12ac^2+16c^4).$$

12. $x^3y^3 - 216z^3$.

$$x^3y^3 - 216z^3 = (xy - 6z)(x^2y^2 + 6xyz + 36z^2).$$

13. $64x^3 - 729y^3$.

$$64x^3 - 729y^3 = (4x - 9y)(16x^2 + 36xy + 81y^2).$$

14. $27a^3 - 512c^3$.

$$27a^3 - 512c^3 = (3a - 8c)(9a^2 + 24ac + 64c^2).$$

15. $8x^6 - 125y^3$.

$$8x^6 - 125y^3 = (2x^2 - 5y)(4x^4 + 10x^2y + 25y^2).$$

16. $64x^{12} - 27y^{15}$.

$$64x^{12} - 27y^{15} = (4x^4 - 3y^5)(16x^8 + 12x^4y^5 + 9y^{10}).$$

17. $216 - 8a^3$.

$$216 - 8a^3 = (6 - 2a)(36 + 12a + 4a^2).$$

18. $343 - 27y^3$.

$$343 - 27y^3 = (7 - 3y)(49 + 21y + 9y^2).$$

Exercise 36. Page 78.

Resolve into factors:

1. $x^3 + 1$.

$$x^3 + 1 = (x + 1)(x^2 - x + 1).$$

2. $8x^3 + y^3$.

$$8x^3 + y^3 = (2x + y)(4x^2 - 2xy + y^2).$$

3. $x^3 + 125$.

$$x^3 + 125 = (x + 5)(x^2 - 5x + 25).$$

4. $64a^3 + 27$.

$$64a^3 + 27 = (4a + 3)(16a^2 - 12a + 9).$$

5. $x^3y^3 + z^3$.

$$x^3y^3 + z^3 = (xy + z)(x^2y^2 - xyz + z^2).$$

6. $a^3 + 64$.

$$a^3 + 64 = (a + 4)(a^2 - 4a + 16).$$

7. $8a^6 + b^3$.

$$8a^6 + b^3 = (2a^2 + b)(4a^4 - 2a^2b + b^2).$$

8. $x^3 + 343$.

$$x^3 + 343 = (x+7)(x^2 - 7x + 49).$$

9. $8 + x^3y^3z^3$.

$$8 + x^3y^3z^3 = (2 + xyz)(4 - 2xyz + x^2y^2z^2).$$

10. $y^9 + 64x^3$.

$$y^9 + 64x^3 = (y^3 + 4x)(y^6 - 4y^3x + 16x^2).$$

11. $a^3b^3 + 27x^3$.

$$a^3b^3 + 27x^3 = (ab + 3x)(a^2b^2 - 3abx + 9x^2).$$

12. $8y^3z^3 + x^6$.

$$8y^3z^3 + x^6 = (2yz + x^2)(4y^2z^2 - 2yzx^2 + x^4).$$

13. $y^9 + 64x^6$.

$$y^9 + 64x^6 = (y^3 + 4x^2)(y^6 - 4y^3x^2 + 16x^4).$$

14. $64a^{12} + x^{15}$.

$$64a^{12} + x^{15} = (4a^4 + x^5)(16a^8 - 4a^4x^5 + x^{10}).$$

15. $27x^{15} + 8a^6$

$$27x^{15} + 8a^6 = (3x^5 + 2a^2)(9x^{10} - 6x^5a^2 + 4a^4).$$

16. $27x^9 + 512$.

$$27x^9 + 512 = (3x^3 + 8)(9x^6 - 24x^3 + 64).$$

17. $343 + 64x^3$.

$$343 + 64x^3 = (7 + 4x)(49 - 28x + 16x^2).$$

18. $125 + 27y^3$.

$$125 + 27y^3 = (5 + 3y)(25 - 15y + 9y^2).$$

Exercise 37. Page 80.

Resolve into factors:

1. $4x^2 + 4xy + y^2$.

$$4x^2 + 4xy + y^2 = (2x + y)(2x + y).$$

2. $x^2 + 6xy + 9y^2$.

$$x^2 + 6xy + 9y^2 = (x + 3y)(x + 3y).$$

3. $x^2 + 16x + 64$.

$$x^2 + 16x + 64 = (x + 8)(x + 8).$$

4. $x^2 + 10ax + 25a^2$.

$$x^2 + 10ax + 25a^2 = (x + 5a)(x + 5a).$$

5. $a^2 - 16a + 64$.

$$a^2 - 16a + 64 = (a - 8)(a - 8).$$

6. $a^2 - 10ab + 25b^2$.

$$a^2 - 10ab + 25b^2 = (a - 5b)(a - 5b).$$

7. $c^2 - 6cd + 9d^2$.

$$c^2 - 6cd + 9d^2 = (c - 3d)(c - 3d).$$

8. $4x^2 - 4x + 1$.

$$4x^2 - 4x + 1 = (2x - 1)(2x - 1).$$

9. $4a^2 - 12ab + 9b^2$.

$$4a^2 - 12ab + 9b^2 = (2a - 3b)(2a - 3b).$$

10. $9a^2 - 24ab + 16b^2$.

$$9a^2 - 24ab + 16b^2 = (3a - 4b)(3a - 4b).$$

11. $x^2 + 8xy + 16y^2$.

$$x^2 + 8xy + 16y^2 = (x + 4y)(x + 4y).$$

12. $x^2 - 8xy + 16y^2$.

$$x^2 - 8xy + 16y^2 = (x - 4y)(x - 4y).$$

13. $4x^2 - 20xy + 25y^2$.

$$4x^2 - 20xy + 25y^2 = (2x - 5y)(2x - 5y).$$

14. $1 + 20a + 100a^2$.

$$1 + 20a + 100a^2 = (1 + 10a)(1 + 10a).$$

15. $49a^2 - 28a + 4$.

$$49a^2 - 28a + 4 = (7a - 2)(7a - 2).$$

16. $36a^2 + 60ab + 25b^2$.

$$36a^2 + 60ab + 25b^2 = (6a + 5b)(6a + 5b).$$

17. $81x^2 - 36bx + 4b^2$.

$$81x^2 - 36bx + 4b^2 = (9x - 2b)(9x - 2b).$$

18. $m^2n^2 + 14mnx^2 + 49x^4$.

$$m^2n^2 + 14mnx + 49x^4 = (mn + 7x^2)(mn + 7x^2).$$

Exercise 38. Page 82.

Resolve into factors:

1. $a^2 + 5a + 6$.

$$a^2 + 5a + 6 = (a + 2)(a + 3).$$

2. $a^2 - 5a + 6$.

$$a^2 - 5a + 6 = (a - 2)(a - 3).$$

3. $a^2 + 6a + 5$.

$$a^2 + 6a + 5 = (a + 1)(a + 5).$$

4. $a^2 - 6a + 5$.

$$a^2 - 6a + 5 = (a - 1)(a - 5).$$

5. $a^2 + 4a - 5$.

$$a^2 + 4a - 5 = (a - 1)(a + 5).$$

6. $a^2 - 4a - 5$.

$$a^2 - 4a - 5 = (a + 1)(a - 5).$$

7. $c^2 - 9c + 18$.

$$c^2 - 9c + 18 = (c - 3)(c - 6).$$

8. $c^2 + 9c + 18$.

$$c^2 + 9c + 18 = (c + 3)(c + 6).$$

9. $c^2 + 3c - 18$.

$$c^2 + 3c - 18 = (c - 3)(c + 6).$$

10. $c^2 - 3c - 18$.

$$c^2 - 3c - 18 = (c + 3)(c - 6).$$

11. $x^2 + 9x + 14$.

$$x^2 + 9x + 14 = (x + 2)(x + 7).$$

12. $x^2 - 9x + 14$.

$$x^2 - 9x + 14 = (x - 2)(x - 7).$$

13. $x^2 - 5x - 14$.

$$x^2 - 5x - 14 = (x + 2)(x - 7).$$

14. $x^2 - 9x + 20$.

$$x^2 - 9x + 20 = (x - 4)(x - 5).$$

15. $x^2 - x - 20$.

$$x^2 - x - 20 = (x + 4)(x - 5).$$

16. $x^2 + x - 20$.

$$x^2 + x - 20 = (x - 4)(x + 5).$$

17. $x^2 - 10x + 21$.

$$x^2 - 10x + 21 = (x - 3)(x - 7).$$

18. $x^2 - 4x - 21$.

$$x^2 - 4x - 21 = (x + 3)(x - 7).$$

19. $x^2 + 4x - 21$.

$$x^2 + 4x - 21 = (x - 3)(x + 7).$$

20. $x^2 - 15x + 56$.

$$x^2 - 15x + 56 = (x - 7)(x - 8).$$

21. $x^2 - x - 56$.

$$x^2 - x - 56 = (x + 7)(x - 8).$$

22. $x^2 - 10x + 9$.

$$x^2 - 10x + 9 = (x - 1)(x - 9).$$

23. $x^2 + 13x + 30$.

$$x^2 + 13x + 30 = (x + 3)(x + 10).$$

24. $x^2 + 7x - 30$.

$$x^2 + 7x - 30 = (x - 3)(x + 10).$$

25. $x^2 - 7x - 30$.

$$x^2 - 7x - 30 = (x + 3)(x - 10).$$

26. $a^2 + ab - 6b^2$.

$$a^2 + ab - 6b^2 = (a - 2b)(a + 3b).$$

27. $a^2 - ab - 6b^2$.

$$a^2 - ab - 6b^2 = (a + 2b)(a - 3b).$$

28. $a^2 + 3ab - 4b^2$.

$$a^2 + 3ab - 4b^2 = (a - b)(a + 4b).$$

29. $a^2 - 3ab - 4b^2$.

$$a^2 - 3ab - 4b^2 = (a + b)(a - 4b).$$

30. $a^2x^2 - 2\,ax - 63.$

$$a^2x^2 - 2\,ax - 63 = (ax + 7)\,(ax - 9).$$

31. $a^2 + 2\,ax - 63\,x^2.$

$$a^2 + 2\,ax - 63\,x^2 = (a - 7\,x)\,(a + 9\,x).$$

32. $a^2 - 9\,ab + 20\,b^2.$

$$a^2 - 9\,ab + 20\,b^2 = (a - 4\,b)\,(a - 5\,b).$$

33. $x^2y^2 - 19\,xyz + 48\,z^2.$

$$x^2y^2 - 19\,xyz + 48\,z^2 = (xy - 3\,z)\,(xy - 16\,z).$$

34. $a^2b^2 + 15\,abc + 44\,c^2.$

$$a^2b^2 + 15\,abc + 44\,c^2. = (ab + 4\,c)\,(ab + 11\,c).$$

35. $x^2 - 13\,xy + 36\,y^2.$

$$x^2 - 13\,xy + 36\,y^2 = (x - 4\,y)\,(x - 9\,y).$$

36. $x^2 + 19\,xy + 84\,y^2.$

$$x^2 + 19\,xy + 84\,y^2 = (x + 7\,y)\,(x + 12\,y).$$

37. $a^2x^2 - 23\,axy + 102\,y^2.$

$$a^2x^2 - 23\,axy + 102\,y^2 = (ax - 6\,y)\,(ax - 17\,y).$$

38. $x^4 - 9\,x^2y^2 + 20\,y^4.$

$$x^4 - 9\,x^2y^2 + 20\,y^4 = (x^2 - 4\,y^2)\,(x^2 - 5\,y^2).$$

39. $a^4x^4 - 24\,a^2x^2y^2 + 143\,y^4.$

$$a^4x^4 - 24\,a^2x^2y^2 + 143\,y^4 = (a^2x^2 - 11\,y^2)\,(a^2x^2 - 13\,y^2).$$

40. $a^6b^6 - 23\,a^3b^3c^2 + 132\,c^4.$

$$a^6b^6 - 23\,a^3b^3c^2 + 132\,c^4 = (a^3b^3 - 11\,c^2)\,(a^3b^3 - 12\,c^2).$$

41. $a^2 - 20\,abc - 96\,b^2c^2.$

$$a^2 - 20\,abc - 96\,b^2c^2 = (a + 4\,bc)\,(a - 24\,bc).$$

42. $a^2 - 4\,abc - 96\,b^2c^2.$

$$a^2 - 4\,abc - 96\,b^2c^2 = (a + 8\,bc)\,(a - 12\,bc).$$

43. $a^2 - 10\,abc - 96\,b^2c^2.$

$$a^2 - 10\,abc - 96\,b^2c^2 = (a + 6\,bc)\,(a - 16\,bc).$$

44. $a^2 + 29\,abc - 96\,b^2c^2.$

$$a^2 + 29\,abc - 96\,b^2c^2 = (a - 3\,bc)\,(a + 32\,bc).$$

45. $a^2 - 46\,abc - 96\,b^2c^2$.

$$a^2 - 46\,abc - 96\,b^2c^2 = (a + 2\,bc)\,(a - 48\,bc).$$

46. $a^2 + 49\,abc + 48\,b^2c^2$.

$$a^2 + 49\,abc + 48\,b^2c^2 = (a + bc)\,(a + 48\,bc).$$

47. $x^2 - 18\,xyz - 243\,y^2z^2$.

$$x^2 - 18\,xyz - 243\,y^2z^2 = (x + 9\,yz)\,(x - 27\,yz).$$

48. $x^2y^2 - xyz - 182\,z^2$.

$$x^2y^2 - xyz - 182\,z^2 = (xy + 13\,z)\,(xy - 14\,z).$$

Exercise 39. Page 83.

Examples for Review.

Resolve into factors:

1. $a^3 - 7\,a$.

$$a^3 - 7\,a = a\,(a^2 - 7).$$

2. $3\,a^2b^2 - 2\,a^3b + 3\,ab^3$.

$$3\,a^2b^2 - 2\,a^3b + 3\,ab^3 = ab\,(3\,ab - 2\,a^2 + 3\,b^2).$$

3. $(a - b)^2 + (a - b)$.

$$\begin{aligned}(a - b)^2 + (a - b) &= (a + b)\,(a - b) + 1\,(a - b)\\ &= (a + b + 1)\,(a - b).\end{aligned}$$

4. $(a + b)^2 - 1$.

$$(a + b)^2 - 1 = (a + b + 1)\,(a + b - 1).$$

5. $a^3 + 8\,b^3$.

$$a^3 + 8\,b^3 = (a + 2\,b)\,(a^2 - 2\,ab + 4\,b^2).$$

6. $(x^2 - 4\,y^2) + (x - 2\,y)$.

$$\begin{aligned}(x^2 - 4\,y^2) + (x - 2\,y) &= (x + 2\,y)\,(x - 2\,y) + 1\,(x - 2\,y)\\ &= (x + 2\,y + 1)\,(x - 2\,y).\end{aligned}$$

7. $(a^3 - b^3) + (a - b)$.

$$\begin{aligned}(a^3 - b^3) + (a - b) &= (a - b)\,(a^2 + ab + b^2) + 1\,(a - b)\\ &= (a - b)\,(a^2 + ab + b^2 + 1).\end{aligned}$$

8. $a^2 - 6\,ab + 9\,b^2$.

$$a^2 - 6\,ab + 9\,b^2 = (a - 3\,b)\,(a - 3\,b).$$

9. $x^2 - x - 2.$

$$x^2 - x - 2 = (x + 1)(x - 2).$$

10. $x^2 - 2x - 3.$

$$x^2 - 2x - 3 = (x + 1)(x - 3).$$

11. $x^2 + 4x - 21.$

$$x^2 + 4x - 21 = (x - 3)(x + 7).$$

12. $a^2 - 11a - 26.$

$$a^2 - 11a - 26 = (a + 2)(a - 13).$$

13. $ax^2 + bx^2 + 3a + 3b.$

$$\begin{aligned} ax^2 + bx^2 + 3a + 3b &= x^2(a + b) + 3(a + b) \\ &= (x^2 + 3)(a + b). \end{aligned}$$

14. $x^2 - 3x - xy + 3y.$

$$\begin{aligned} x^2 - 3x - xy + 3y &= x(x - y) - 3(x - y) \\ &= (x - 3)(x - y). \end{aligned}$$

15. $x^2 - 7x + 12.$

$$x^2 - 7x + 12 = (x - 3)(x - 4).$$

16. $a^2 + 5ab + 6b^2.$

$$a^2 + 5ab + 6b^2 = (a + 2b)(a + 3b).$$

17. $x^4 + 10x^2 + 25.$

$$x^4 + 10x^2 + 25 = (x^2 + 5)(x^2 + 5).$$

18. $x^2 - 18x + 81.$

$$x^2 - 18x + 81 = (x - 9)(x - 9).$$

19. $x^2 - 21x + 110.$

$$x^2 - 21x + 110 = (x - 10)(x - 11).$$

20. $x^2 + 19x + 88.$

$$x^2 + 19x + 88 = (x + 8)(x + 11).$$

21. $x^2 - 19x + 88.$

$$x^2 - 19x + 88 = (x - 8)(x - 11).$$

22. $x^3 - x^2 + x - 1.$

$$\begin{aligned} x^3 - x^2 + x - 1 &= x^2(x - 1) + 1(x - 1) \\ &= (x^2 + 1)(x - 1). \end{aligned}$$

23. $9x^4 - x^2.$

$$\begin{aligned} 9x^4 - x^2 &= x^2(9x^2 - 1) \\ &= x^2(3x+1)(3x-1). \end{aligned}$$

24. $1 - (a-b)^2.$

$$1-(a-b)^2 = (1+a-b)(1-a+b).$$

25. $(a^3+b^3)+(a+b).$

$$\begin{aligned} (a^3+b^3)+(a+b) &= (a+b)(a^2-ab+b^2)+1(a+b) \\ &= (a+b)(a^2-ab+b^2+1). \end{aligned}$$

26. $m^2x - n^2x + m^2y - n^2y.$

$$\begin{aligned} m^2x - n^2x + m^2y - n^2y &= m^2(x+y) - n^2(x+y) \\ &= (m^2-n^2)(x+y) \\ &= (m+n)(m-n)(x+y). \end{aligned}$$

27. $(x-y)^2 - z^2.$

$$(x-y)^2 - z^2 = (x-y+z)(x-y-z).$$

28. $z^2 - (x-y)^2.$

$$z^2-(x-y)^2 = (z+x-y)(z-x+y).$$

29. $4a^4 - (3a-1)^2.$

$$4a^4-(3a-1)^2 = (2a^2+3a-1)(2a^2-3a+1).$$

30. $8x^3 - y^3.$

$$8x^3-y^3 = (2x-y)(4x^2-2xy+y^2).$$

31. $x^3 - 3x^2y.$

$$x^3-3x^2y = x^2(x-3y).$$

32. $x^3 - 27y^3.$

$$x^3-27y^3 = (x-3y)(x^2+3xy+9y^2).$$

33. $x^2+3x-40.$

$$x^2+3x-40 = (x-5)(x+8).$$

34. $x^2+3xy-10y^2.$

$$x^2+3xy-10y^2 = (x-2y)(x+5y).$$

35. $1-16x^2.$

$$1-16x^2 = (1+4x)(1-4x).$$

36. $a^6 - 9a^2b^4.$

$$\begin{aligned} a^6 - 9a^2b^4 &= a^2(a^4-9b^4). \\ &= a^2(a^2+3b^2)(a^2-3b^2). \end{aligned}$$

37. $x^3 + 3x^2y + 2xy^2$.

$$x^3 + 3x^2y + 2xy^2 = x(x^2 + 3xy + 2y^2)$$
$$= x(x+y)(x+2y).$$

38. $x^4 + 4x^3y + 3x^2y^2$.

$$x^4 + 4x^3y + 3x^2y^2 = x^2(x^2 + 4xy + 3y^2)$$
$$= x^2(x+y)(x+3y).$$

39. $x^2 - 4xy^2 + 4y^4$.

$$x^2 - 4xy^2 + 4y^4 = (x - 2y^2)(x - 2y^2).$$

40. $16x^4 + 8x^2 + 1$.

$$16x^4 + 8x^2 + 1 = (4x^2 + 1)(4x^2 + 1).$$

41. $9a^4 - 4a^2c^2$.

$$9a^4 - 4a^2c^2 = a^2(9a^2 - 4c^2)$$
$$= a^2(3a + 2c)(3a - 2c).$$

42. $a^3b - a^2b^2 - 2ab^3$.

$$a^3b - a^2b^2 - 2ab^3 = ab(a^2 - ab - 2b^2)$$
$$= ab(a+b)(a-2b).$$

43. $x^4 - x^3 + 8x - 8$.

$$x^4 - x^3 + 8x - 8 = x^3(x-1) + 8(x-1)$$
$$= (x^3 + 8)(x-1)$$
$$= (x+2)(x^2 - 2x + 4)(x-1).$$

44. $a^4 - a^3x + ay^3 - xy^3$.

$$a^4 - a^3x + ay^3 - xy^3 = a^3(a-x) + y^3(a-x)$$
$$= (a^3 + y^3)(a-x)$$
$$= (a+y)(a^2 - ay + y^2)(a-x).$$

Exercise 40. Page 86.

Find the H. C. F. of:

1. 330 and 546.

$$330 = 2 \times 3 \times 5 \times 11$$
$$546 = 2 \times 3 \times 7 \times 13.$$
$$\therefore \text{the H. C. F.} = 2 \times 3 = 6.$$

2. $20x^3$ and $15x^4$.

$$20x^3 = 2 \times 2 \times 5 \times x^3$$
$$15x^4 = 3 \times 5 \times x^4.$$
$$\therefore \text{the H. C. F.} = 5x^3.$$

3. $42\,ax^2$ and $60\,a^2x$.

$$\begin{aligned} 42\,ax^2 &= 2 \times 3 \times 7 \times a \times x^2 \\ 60\,a^2x &= 2 \times 2 \times 3 \times 5 \times a^2 \times x. \\ \therefore \text{the H. C. F.} &= 2 \times 3 \times a \times x \\ &= 6\,ax. \end{aligned}$$

4. $35\,a^2b^2$ and $49\,ab^3$.

$$\begin{aligned} 35\,a^2b^2 &= 5 \times 7 \times a^2 \times b^2 \\ 49\,ab^3 &= 7 \times 7 \times a \times b^3. \\ \therefore \text{the H. C. F.} &= 7\,ab^2. \end{aligned}$$

5. $28\,x^4$ and $63\,y^4$.

$$\begin{aligned} 28\,x^4 &= 2 \times 2 \times 7 \times x^4 \\ 63\,y^4 &= 3 \times 3 \times 7 \times y^4. \\ \therefore \text{the H. C. F.} &= 7. \end{aligned}$$

6. $54\,a^2b^2$ and $56\,a^3b^3$.

$$\begin{aligned} 54\,a^2b^2 &= 2 \times 3 \times 3 \times 3 \times a^2 \times b^2 \\ 56\,a^3b^3 &= 2 \times 2 \times 2 \times 7 \times a^3 \times b^3. \\ \therefore \text{the H. C. F.} &= 2\,a^2b^2. \end{aligned}$$

7. $x^3 + 3\,x^2y$ and $x^3 + 27\,y^3$.

$$\begin{aligned} x^3 + 3\,x^2y &= x^2\,(x + 3\,y) \\ x^3 + 27\,y^3 &= (x + 3\,y)\,(x^2 - 3\,xy + 9\,y^2). \\ \therefore \text{the H. C. F.} &= x + 3\,y. \end{aligned}$$

8. $x^2 + 3x$ and $x^2 - 9$.

$$\begin{aligned} x^2 + 3\,x &= x\,(x + 3) \\ x^2 - 9 &= (x + 3)\,(x - 3). \\ \therefore \text{the H. C. F.} &= x + 3. \end{aligned}$$

9. $2\,ax^3 + x^3$ and $8\,a^3 + 1$.

$$\begin{aligned} 2\,ax^3 + x^3 &= x^3\,(2\,a + 1) \\ 8\,a^3 + 1 &= (2\,a + 1)\,(4\,a^2 - 2\,a + 1). \\ \therefore \text{the H. C. F.} &= 2\,a + 1. \end{aligned}$$

10. $(x + y)^2$ and $x^2 - y^2$.

$$\begin{aligned} (x + y)^2 &= (x + y)\,(x + y) \\ x^2 - y^2 &= (x + y)\,(x - y). \\ \therefore \text{the H. C. F.} &= x + y. \end{aligned}$$

11. $a^3 + a^2x$ and $a^2 - x^2$.

$$a^3 + a^2x = a^2(a + x)$$
$$a^2 - x^2 = (a + x)(a - x).$$
$$\therefore \text{the H. C. F.} = a + x.$$

12. $a^2 - 4b^2$ and $a^2 + 2ab$.

$$a^2 - 4b^2 = (a + 2b)(a - 2b)$$
$$a^2 + 2ab = a(a + 2b).$$
$$\therefore \text{the H. C. F.} = a + 2b.$$

13. $x^2 - 1$ and $x^2 + 2x - 3$.

$$x^2 - 1 = (x + 1)(x - 1)$$
$$x^2 + 2x - 3 = (x - 1)(x + 3).$$
$$\therefore \text{the H. C. F.} = x - 1.$$

14. $x^2 + 5x + 6$ and $x^2 + 4x + 3$.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$
$$x^2 + 4x + 3 = (x + 1)(x + 3).$$
$$\therefore \text{the H. C. F.} = x + 3.$$

15. $x^2 - 9x + 18$ and $x^2 - 10x + 24$.

$$x^2 - 9x + 18 = (x - 3)(x - 6)$$
$$x^2 - 10x + 24 = (x - 4)(x - 6).$$
$$\therefore \text{the H. C. F.} = x - 6.$$

16. $x^3 + 1$ and $x^2 - x + 1$.

$$x^3 + 1 = (x + 1)(x^2 - x + 1)$$
$$x^2 - x + 1 = x^2 - x + 1.$$
$$\therefore \text{the H. C. F.} = x^2 - x + 1.$$

17. $x^2 - 3x + 2$ and $x^2 - 4x + 3$.

$$x^2 - 3x + 2 = (x - 1)(x - 2)$$
$$x^2 - 4x + 3 = (x - 1)(x - 3).$$
$$\therefore \text{the H. C. F.} = x - 1.$$

18. $x^2 - 3xy + 2y^2$ and $x^2 - 2xy + y^2$.

$$x^2 - 3xy + 2y^2 = (x - y)(x - 2y)$$
$$x^2 - 2xy + y^2 = (x - y)(x - y).$$
$$\therefore \text{the H. C. F.} = x - y.$$

19. $x^2 - 4x - 5$ and $x^2 - 25$.

$$x^2 - 4x - 5 = (x + 1)(x - 5)$$
$$x^2 - 25 = (x - 5)(x - 5).$$
$$\therefore \text{ the H. C. F.} = x - 5.$$

20. $(a - b)^2 - c^2$ and $ab - b^2 - bc$.

$$(a - b)^2 - c^2 = (a - b + c)(a - b - c)$$
$$ab - b^2 - bc = b(a - b - c).$$
$$\therefore \text{ the H. C. F.} = a - b - c.$$

21. $x^2 + xy - 2y^2$ and $x^2 + 5xy + 6y^2$.

$$x^2 + xy - 2y^2 = (x - y)(x + 2y)$$
$$x^2 + 5xy + 6y^2 = (x + 2y)(x + 3y).$$
$$\therefore \text{ the H. C. F.} = x + 2y.$$

22. $x^2 + 7xy + 12y^2$ and $x^2 + 3xy - 4y^2$.

$$x^2 + 7xy + 12y^2 = (x + 3y)(x + 4y)$$
$$x^2 + 3xy - 4y^2 = (x - y)(x + 4y).$$
$$\therefore \text{ the H. C. F.} = x + 4y.$$

23. $x^3 - 8y^3$ and $x^2 + 2xy + 4y^2$.

$$x^3 - 8y^3 = (x - 2y)(x^2 + 2xy + 4y^2)$$
$$x^2 + 2xy + 4y^2 = x^2 + 2xy + 4y^2.$$
$$\therefore \text{ the H. C. F.} = x^2 + 2xy + 4y^2.$$

24. $x^3 - 2x^2 - x + 2$ and $x^2 - 4x + 4$.

$$\begin{aligned} x^3 - 2x^2 - x + 2 &= x^2(x - 2) - 1(x - 2) \\ &= (x^2 - 1)(x - 2) \\ &= (x + 1)(x - 1)(x - 2) \end{aligned}$$
$$x^2 - 4x + 4 = (x - 2)(x - 2).$$
$$\therefore \text{ the H. C. F.} = x - 2.$$

25. $1 - 5a + 6a^2$ and $1 - 7a + 12a^2$.

$$1 - 5a + 6a^2 = (1 - 2a)(1 - 3a)$$
$$1 - 7a + 12a^2 = (1 - 3a)(1 - 4a).$$
$$\therefore \text{ the H. C. F.} = 1 - 3a.$$

26. $x^2 - 8xy + 7y^2$ and $x^2 - 3xy - 28y^2$.

$$x^2 - 8xy + 7y^2 = (x - y)(x - 7y)$$
$$x^2 - 3xy - 28y^2 = (x + 4y)(x - 7y).$$
$$\therefore \text{ the H. C. F.} = x - 7y.$$

27. $8a^3 + b^3$ and $4a^2 + 4ab + b^2$.

$$8a^3 + b^3 = (2a + b)(4a^2 - 2ab + b^2)$$
$$4a^2 + 4ab + b^2 = (2a + b)(2a + b).$$
$$\therefore \text{ the H. C. F.} = 2a + b.$$

28. $x^2 - (y - z)^2$ and $(x + y)^2 - z^2$.

$$x^2 - (y - z)^2 = (x + y - z)(x - y + z)$$
$$(x + y)^2 - z^2 = (x + y + z)(x + y - z).$$
$$\therefore \text{ the H. C. F.} = x + y - z.$$

Exercise 41. Page 88.

Find the L. C. M. of:

1. $9xy^3$ and $6x^2y$.

$$9xy^3 = 3 \times 3 \times x \times y^3$$
$$6x^2y = 2 \times 3 \times x^2 \times y.$$
$$\therefore \text{ the L. C. M.} = 2 \times 3 \times 3 \times x^2 \times y^3$$
$$= 18x^2y^3.$$

2. $3abc^2$ and $2a^2bc^3$.

$$3abc^2 = 3 \times a \times b \times c^2$$
$$2a^2bc^3 = 2 \times a^2 \times b \times c^3.$$
$$\therefore \text{ the L. C. M.} = 2 \times 3 \times a^2 \times b \times c^3$$
$$= 6a^2bc^3.$$

3. $4a^3b$ and $10ab^3$.

$$4a^3b = 2 \times 2 \times a^3 \times b$$
$$10ab^3 = 2 \times 5 \times a \times b^3.$$
$$\therefore \text{ the L. C. M.} = 2 \times 2 \times 5 \times a^3 \times b^3$$
$$= 20a^3b^3.$$

4. $6a^3b^3$ and $15a^2b^4$.

$$6a^3b^3 = 2 \times 3 \times a^3 \times b^3$$
$$15a^2b^4 = 3 \times 5 \times a^2 \times b^4.$$
$$\therefore \text{ the L. C. M.} = 2 \times 3 \times 5 \times a^3 \times b^4$$
$$= 30a^3b^4.$$

5. $21xy^3$ and $27x^3y^5$.

$$21xy^3 = 3 \times 7 \times x \times y^3$$
$$27x^3y^5 = 3 \times 3 \times 3 \times x^3 \times y^5.$$
$$\therefore \text{ the L. C. M.} = 3 \times 3 \times 3 \times 7 \times x^3 \times y^5$$
$$= 189x^3y^5.$$

6. xy^3z^2 and $x^2y^2z^3$.

$$xy^3z^2 = x \times y^3 \times z^2$$
$$x^2y^2z^3 = x^2 \times y^2 \times z^3.$$
$$\therefore \text{the L. C. M.} = x^2y^3z^3.$$

7. a^2 and $a^2 + a$.

$$a^2 = a^2$$
$$a^2 + a = a(a+1).$$
$$\therefore \text{the L. C. M.} = a^2(a+1).$$

8. x^2 and $x^3 - 3x^2$.

$$x^2 = x^2$$
$$x^3 - 3x^2 = x^2(x-3).$$
$$\therefore \text{the L. C. M.} = x^2(x-3).$$

9. $x^2 - 1$ and $x^2 + x$.

$$x^2 - 1 = (x+1)(x-1)$$
$$x^2 + x = x(x+1).$$
$$\therefore \text{the L. C. M.} = x(x+1)(x-1).$$

10. $x^2 - 1$ and $x^2 - x$.

$$x^2 - 1 = (x+1)(x-1)$$
$$x^2 - x = x(x-1).$$
$$\therefore \text{the L. C. M.} = x(x+1)(x-1).$$

11. $x^2 + xy$ and $xy + y^2$.

$$x^2 + xy = x(x+y)$$
$$xy + y^2 = y(x+y).$$
$$\therefore \text{the L. C. M.} = xy(x+y).$$

12. $x^2 + 2x$ and $(x+2)^2$.

$$x^2 + 2x = x(x+2)$$
$$(x+2)^2 = (x+2)^2.$$
$$\therefore \text{the L. C. M.} = x(x+2)^2.$$

13. $a^2 + 4a + 4$ and $a^2 + 5a + 6$.

$$a^2 + 4a + 4 = (a+2)(a+2)$$
$$a^2 + 5a + 6 = (a+2)(a+3).$$
$$\therefore \text{the L. C. M.} = (a+2)(a+2)(a+3).$$

14. $c^2 + c - 20$ and $c^2 - c - 30$.

$$c^2 + c - 20 = (c - 4)(c + 5)$$
$$c^2 - c - 30 = (c + 5)(c - 6).$$
$$\therefore \text{the L. C. M.} = (c - 4)(c + 5)(c - 6).$$

15. $b^2 + b - 42$ and $b^2 - 11b + 30$.

$$b^2 + b - 42 = (b - 6)(b + 7)$$
$$b^2 - 11b + 30 = (b - 5)(b - 6).$$
$$\therefore \text{the L. C. M.} = (b - 5)(b - 6)(b + 7).$$

16. $y^2 - 10y + 24$ and $y^2 + y - 20$.

$$y^2 - 10y + 24 = (y - 4)(y - 6)$$
$$y^2 + y - 20 = (y - 4)(y + 5).$$
$$\therefore \text{the L. C. M.} = (y - 4)(y + 5)(y - 6).$$

17. $z^2 + 2z - 35$ and $z^2 - 11z + 30$.

$$z^2 + 2z - 35 = (z - 5)(z + 7)$$
$$z^2 - 11z + 30 = (z - 5)(z - 6).$$
$$\therefore \text{the L. C. M.} = (z - 5)(z - 6)(z + 7).$$

18. $x^2 - 64$; $x^3 - 64$; and $x + 8$.

$$x^2 - 64 = (x + 8)(x - 8)$$
$$x^3 - 64 = (x - 4)(x^2 + 4x + 16)$$
$$x + 8 = x + 8.$$
$$\therefore \text{the L. C. M.} = (x - 4)(x + 8)(x - 8)(x^2 + 4x + 16).$$

19. $a^2 - b^2$; $(a + b)^2$; and $(a - b)^2$.

$$a^2 - b^2 = (a + b)(a - b)$$
$$(a + b)^2 = (a + b)(a + b)$$
$$(a - b)^2 = (a - b)(a - b).$$
$$\therefore \text{the L. C. M.} = (a + b)(a + b)(a - b)(a - b).$$

20. $4ab(a + b)^2$ and $2a^2(a^2 - b^2)$.

$$4ab(a + b)^2 = 4ab(a + b)(a + b)$$
$$2a^2(a^2 - b^2) = 2a^2(a + b)(a - b).$$
$$\therefore \text{the L. C. M.} = 4a^2b(a + b)(a + b)(a - b).$$

21. $y^2 + 7y + 12$; $y^2 + 6y + 8$; and $y^2 + 5y + 6$.

$$y^2 + 7y + 12 = (y + 3)(y + 4)$$
$$y^2 + 6y + 8 = (y + 2)(y + 4)$$
$$y^2 + 5y + 6 = (y + 2)(y + 3).$$
$$\therefore \text{the L. C. M.} = (y + 2)(y + 3)(y + 4).$$

22. $x^2 - 1$; $x^3 + x^2 + x + 1$; and $x^3 - x^2 + x - 1$.

$$x^2 - 1 = (x + 1)(x - 1)$$
$$x^3 + x^2 + x + 1 = x^2(x + 1) + 1(x + 1)$$
$$= (x^2 + 1)(x + 1)$$
$$x^3 - x^2 + x - 1 = x^2(x - 1) + 1(x - 1)$$
$$= (x^2 + 1)(x - 1).$$
$$\therefore \text{the L. C. M.} = (x + 1)(x - 1)(x^2 + 1).$$

23. $1 - x^2$; $1 - x^3$; and $1 + x$.

$$1 - x^2 = (1 + x)(1 - x)$$
$$1 - x^3 = (1 - x)(1 + x + x^2)$$
$$1 + x = 1 + x.$$
$$\therefore \text{the L. C. M.} = (1 + x)(1 - x)(1 + x + x^2).$$

24. $x^2 + 2xy + y^2$; $x^2 - y^2$; and $x^2 - 2xy + y^2$.

$$x^2 + 2xy + y^2 = (x + y)(x + y)$$
$$x^2 - y^2 = (x + y)(x - y)$$
$$x^2 - 2xy + y^2 = (x - y)(x - y).$$
$$\therefore \text{the L. C. M.} = (x + y)(x + y)(x - y)(x - y).$$

25. $x^3 - 27$; $x^2 + 2x - 15$; $x^2 + 5x$.

$$x^3 - 27 = (x - 3)(x^2 + 3x + 9)$$
$$x^2 + 2x - 15 = (x - 3)(x + 5)$$
$$x^2 + 5x = x(x + 5).$$
$$\therefore \text{the L. C. M.} = x(x - 3)(x + 5)(x^2 + 3x + 9).$$

26. $(a + b)^2 - c^2$; $(a + b + c)^2$; and $a + b - c$.

$$(a + b)^2 - c^2 = (a + b + c)(a + b - c)$$
$$(a + b + c)^2 = (a + b + c)(a + b + c)$$
$$(a + b - c) = (a + b - c).$$
$$\therefore \text{the L. C. M.} = (a + b + c)(a + b + c)(a + b - c).$$

27. $x^2 - (a + b)x + ab$ and $x^2 - (a + c)x + ac$.

$$x^2 - (a + b)x + ab = (x - a)(x - b)$$
$$x^2 - (a + c)x + ac = (x - a)(x - c).$$
$$\therefore \text{the L. C. M.} = (x - a)(x - b)(x - c).$$

28. $(a + b)^2 - c^2$ and $a^2 + ab + ac$.

$$(a + b)^2 - c^2 = (a + b + c)(a + b - c)$$
$$a^2 + ab + ac = a(a + b + c).$$
$$\therefore \text{the L. C. M.} = a(a + b + c)(a + b - c).$$

Exercise 42. Page 90.

Reduce to lowest terms:

1. $\dfrac{2a}{6ab}$.

$$\frac{2a}{6ab}=\frac{2a}{2a\times 3b}=\frac{1}{3b}.$$

2. $\dfrac{12m^2n}{15mn^2}$.

$$\frac{12m^2n}{15mn^2}=\frac{3mn\times 4m}{3mn\times 5n}=\frac{4m}{5n}.$$

3. $\dfrac{21m^2p^2}{28mp^4}$.

$$\frac{21m^2p^2}{28mp^4}=\frac{7mp^2\times 3m}{7mp^2\times 4p^2}=\frac{3m}{4p^2}.$$

4. $\dfrac{3x^3y^2z}{6xy^3z^2}$.

$$\frac{3x^3y^2z}{6xy^3z^2}=\frac{3xy^2z\times x^2}{3xy^2z\times 2yz}=\frac{x^2}{2yx}.$$

5. $\dfrac{5a^3b^3c^3}{15c^5}$.

$$\frac{5a^3b^3c^3}{15c^5}=\frac{5c^3\times a^3b^3}{5c^3\times 3c^2}=\frac{a^3b^3}{3c^2}.$$

6. $\dfrac{34x^3y^4z^5}{51x^2y^3z^5}$.

$$\frac{34x^3y^4z^5}{51x^2y^3z^5}=\frac{17x^2y^3z^5\times 2xy}{17x^2y^3z^5\times 3}=\frac{2xy}{3}.$$

7. $\dfrac{46m^2np^3}{69mnp^4}$.

$$\frac{46m^2np^3}{69mnp^4}=\frac{23mnp^3\times 2m}{23mnp^3\times 3p}=\frac{2m}{3p}.$$

8. $\dfrac{39a^2b^3c^4}{52a^5bc^3}$.

$$\frac{39a^2b^3c^4}{52a^5bc^3}=\frac{13a^2bc^3\times 3b^2c}{13a^2bc^3\times 4a^3}=\frac{3b^2c}{4a^3}.$$

9. $\dfrac{58xy^4z^6}{87xy^2z^2}$.

$$\frac{58xy^4z^6}{87xy^2z^2}=\frac{29xy^2z^2\times 2y^2z^4}{29xy^2z^2\times 3}=\frac{2y^2z^4}{3}.$$

10. $\dfrac{abx-bx^2}{acx-cx^2}$.

$$\frac{abx-bx^2}{acx-cx^2}=\frac{bx(a-x)}{cx(a-x)}=\frac{b}{c}.$$

11. $\dfrac{4a^2-9b^2}{4a^2+6ab}$.

$$\frac{4a^2-9b^2}{4a^2+6ab}=\frac{(2a+3b)(2a-3b)}{2a(2a+3b)}=\frac{2a-3b}{2a}.$$

12. $\dfrac{3a^2+6a}{a^2+4a+4}$.

$$\frac{3a^2+6a}{a^2+4a+4}=\frac{3a(a+2)}{(a+2)(a+2)}=\frac{3a}{a+2}.$$

13. $\dfrac{x^2+5x}{x^2+4x-5}$.

$$\frac{x^2+5x}{x^2+4x-5}=\frac{x(x+5)}{(x-1)(x+5)}=\frac{x}{x-1}.$$

14. $\dfrac{xy-3y^2}{x^3-27y^3}$.

$$\frac{xy-3y^2}{x^3-27y^3}=\frac{y(x-3y)}{(x-3y)(x^2+3xy+9y^2)}=\frac{y}{x^2+3xy+9y^2}.$$

15. $\dfrac{x^2+5x+4}{x^2-x-20}$.

$$\frac{x^2+5x+4}{x^2-x-20}=\frac{(x+1)(x+4)}{(x+4)(x-5)}=\frac{x+1}{x-5}.$$

16. $\dfrac{x^2+2x+1}{x^2-x-1}$.

$$\frac{x^2+2x+1}{x^2-x-1}=\frac{(x+1)(x+1)}{(x+1)(x-2)}=\frac{x+1}{x-2}.$$

17. $\dfrac{(a+b)^2-c^2}{a^2+ab-ac}$.

$$\frac{(a+b)^2-c^2}{a^2+ab-ac}=\frac{(a+b+c)(a+b-c)}{a(a+b-c)}=\frac{a+b+c}{a}.$$

18. $\dfrac{x^2+9x+20}{x^2+7x+12}$.

$$\frac{x^2+9x+20}{x^2+7x+12}=\frac{(x+4)(x+5)}{(x+3)(x+4)}=\frac{x+5}{x+3}.$$

19. $\dfrac{x^2-14x-15}{x^2-12x-45}$.

$$\frac{x^2-14x-15}{x^2-12x-45}=\frac{(x+1)(x-15)}{(x+3)(x-15)}=\frac{x+1}{x+3}.$$

Exercise 43. Page 91.

Reduce to integral or mixed expressions:

1. $\dfrac{a^2-b^2+2}{a-b}$.

$$\begin{array}{l|l}
a^2 \qquad -b^2+2 & a-b \\ \cline{2-2}
\underline{a^2-ab\qquad\quad} & a+b \\ \cline{1-1}
\quad ab-b^2+2 & \\
\quad \underline{ab-b^2\qquad} & \\
\qquad\qquad\quad 2 &
\end{array}$$

$$=a+b+\frac{2}{a-b}.$$

2. $\dfrac{a^2-b^2-2}{a+b}$.

$$\begin{array}{l|l}
a^2 \qquad -b^2-2 & a+b \\ \cline{2-2}
\underline{a^2+ab\qquad\quad} & a-b \\ \cline{1-1}
\quad -ab-b^2-2 & \\
\quad \underline{-ab-b^2\qquad} & \\
\qquad\qquad\quad -2 &
\end{array}$$

$$=a-b-\frac{2}{a+b}.$$

3. $\dfrac{a^3-2a^2+2a+1}{a^2-a-1}$.

$$\begin{array}{r|l}
a^3-2a^2+2a+1 & a^2-a-1 \\ \cline{2-2}
a^3-a^2-a & a-1 \\ \cline{1-1}
-a^2+3a+1 & \\
-a^2+a+1 & \\ \cline{1-1}
2a &
\end{array}$$

$$= a-1+\frac{2a}{a^2-a-1}.$$

4. $\dfrac{2x^2-2x+1}{x+1}$.

$$\begin{array}{r|l}
2x^2-2x+1 & x+1 \\ \cline{2-2}
2x^2+2x & 2x-4 \\ \cline{1-1}
-4x+1 & \\
-4x-4 & \\ \cline{1-1}
5 &
\end{array}$$

$$= 2x-4+\frac{5}{x+1}.$$

5. $\dfrac{8x^3}{2x+1}$.

$$\begin{array}{r|l}
8x^3 & 2x+1 \\ \cline{2-2}
8x^3+4x^2 & 4x^2-2x+1 \\ \cline{1-1}
-4x^2 & \\
-4x^2-2x & \\ \cline{1-1}
2x & \\
2x+1 & \\ \cline{1-1}
-1 &
\end{array}$$

$$= 4x^2-2x+1-\frac{1}{2x+1}.$$

6. $\dfrac{5x^3+9x^2+3}{x^2+x-1}$.

$$\begin{array}{r|l}
5x^3+9x^2+3 & x^2+x-1 \\ \cline{2-2}
5x^3+5x^2-5x & 5x+4 \\ \cline{1-1}
4x^2+5x+3 & \\
4x^2+4x-4 & \\ \cline{1-1}
x+7 &
\end{array}$$

$$= 5x+4+\frac{x+7}{x^2+x-1}.$$

7. $\dfrac{a^3+a^2+7a-2}{a^2+a+2}$.

$$\begin{array}{l|l}
a^3+a^2+7a-2 & a^2+a+2 \\ \cline{2-2}
\underline{a^3+a^2+2a} & a \\
\qquad\qquad 5a-2 &
\end{array}$$

$$=a+\frac{5a-2}{a^2+a+2}.$$

8. $\dfrac{y^4+y^2x^2+x^4}{y^2+yx+x^2}$.

$$\begin{array}{l|l}
y^4 \qquad\quad +y^2x^2 \qquad\quad +x^4 & y^2+yx+x^2 \\ \cline{2-2}
\underline{y^4+y^3x+y^2x^2} & y^2-yx+x^2 \\
\quad -y^3x \qquad\qquad\qquad +x^4 & \\
\quad \underline{-y^3x-y^2x^2-yx^3} & \\
\qquad\qquad y^2x^2+yx^3+x^4 & \\
\qquad\qquad \underline{y^2x^2+yx^3+x^4} &
\end{array}$$

$$=y^2-yx+x^2.$$

9. $\dfrac{x^4-3x^3+x-1}{x^2+x+1}$.

$$\begin{array}{l|l}
x^4-3x^3 \qquad\quad + \quad x-1 & x^2+x+1 \\ \cline{2-2}
\underline{x^4+\ \ x^3+\ \ x^2} & x^2-4x+3 \\
\quad -4x^3-\ \ x^2+\ \ x-1 & \\
\quad \underline{-4x^3-4x^2-4x} & \\
\qquad\qquad 3x^2+5x-1 & \\
\qquad\qquad \underline{3x^2+3x+3} & \\
\qquad\qquad\qquad 2x-4 &
\end{array}$$

$$=x^2-4x+3+\frac{2x-4}{x^2+x+1}.$$

10. $\dfrac{x^5-x^4+1}{x^2-x-1}$.

$$\begin{array}{l|l}
x^5-x^4 \qquad\qquad\qquad\quad +1 & x^2-x-1 \\ \cline{2-2}
\underline{x^5-x^4-x^3} & x^3+x+1 \\
\qquad\qquad x^3 \qquad\qquad +1 & \\
\qquad\qquad \underline{x^3-x^2-\ \ x} & \\
\qquad\qquad\quad x^2+\ \ x+1 & \\
\qquad\qquad\quad \underline{x^2-\ \ x-1} & \\
\qquad\qquad\qquad 2x+2 &
\end{array}$$

$$=x^3+x+1+\frac{2x+2}{x^2-x-1}.$$

Exercise 44. Page 92.

Reduce to a fraction:

1. $x-y+\dfrac{2xy}{x-y}$.

$$x-y+\frac{2xy}{x-y}=\frac{(x-y)(x-y)+2xy}{x-y}$$
$$=\frac{x^2-2xy+y^2+2xy}{x-y}$$
$$=\frac{x^2+y^2}{x-y}.$$

2. $x+y-\dfrac{2xy}{x+y}$.

$$x+y-\frac{2xy}{x+y}=\frac{(x+y)(x+y)-2xy}{x+y}$$
$$=\frac{x^2+2xy+y^2-2xy}{x+y}$$
$$=\frac{x^2+y^2}{x+y}.$$

3. $1-\dfrac{x-y}{x+y}$.

$$1-\frac{x-y}{x+y}=\frac{(x+y)-(x-y)}{x+y}$$
$$=\frac{x+y-x+y}{x+y}$$
$$=\frac{2y}{x+y}.$$

4. $a-x-\dfrac{a^2+x^2}{a-x}$.

$$a-x-\frac{a^2+x^2}{a-x}=\frac{(a-x)(a-x)-(a^2+x^2)}{a-x}$$
$$=\frac{a^2-2ax+x^2-a^2-x^2}{a-x}$$
$$=-\frac{2ax}{a-x}.$$

5. $x+2-\frac{x^2-4}{x-3}$.

$$x+2-\frac{x^2-4}{x-3}=\frac{(x+2)(x-3)-(x^2-4)}{x-3}$$
$$=\frac{x^2-x-6-x^2+4}{x-3}$$
$$=\frac{-x-2}{x-3}$$
$$=-\frac{x+2}{x-3}.$$

6. $\frac{x-3}{x-2}-2x+1$.

$$\frac{x-3}{x-2}-2x+1=\frac{x-3-(2x-1)(x-2)}{x-2}$$
$$=\frac{x-3-(2x^2-5x+2)}{x-2}$$
$$=\frac{x-3-2x^2+5x-2}{x-2}$$
$$=-\frac{2x^2-6x+5}{x-2}.$$

7. $\frac{x+3}{x+2}+x^2-x-1$.

$$\frac{x+3}{x+2}+x^2-x-1=\frac{x+3+(x^2-x-1)(x+2)}{x+2}$$
$$=\frac{x+3+x^3+x^2-3x-2}{x+2}$$
$$=\frac{x^3+x^2-2x+1}{x+2}.$$

8. $2a-1+\frac{3-4a}{a-3}$.

$$2a-1+\frac{3-4a}{a-3}=\frac{(2a-1)(a-3)+3-4a}{a-3}$$
$$=\frac{2a^2-7a+3+3-4a}{a-3}$$
$$=\frac{2a^2-11a+6}{a-3}.$$

9. $1-2a^2-\dfrac{a^2-a+2}{a-1}$.

$$1-2a^2-\frac{a^2-a+2}{a-1}=\frac{(1-2a^2)(a-1)-(a^2-a+2)}{a-1}$$
$$=\frac{-2a^3+2a^2+a-1-a^2+a-2}{a-1}$$
$$=\frac{-2a^3+a^2+2a-3}{a-1}.$$

10. $a^2+2a-5-\dfrac{2a-1}{3a^2+1}$.

$$a^2+2a-5-\frac{2a-1}{3a^2+1}=\frac{(a^2+2a-5)(3a^2+1)-(2a-1)}{3a^2+1}$$
$$=\frac{3a^4+6a^3-14a^2+2a-5-2a+1}{3a^2+1}$$
$$=\frac{3a^4+6a^3-14a^2-4}{3a^2+1}.$$

Exercise 45. Page 94.

Express with lowest common denominator:

1. $\dfrac{x}{x-a},\ \dfrac{x^2}{x^2-a^2}$.

$$\frac{x}{x-a}=\frac{x}{x-a}$$
$$\frac{x^2}{x^2-a^2}=\frac{x^2}{(x+a)(x-a)}.$$
$\therefore$ the L. C. D. $=(x+a)(x-a)$
$$\frac{x}{x-a}=\frac{x(x+a)}{(x+a)(x-a)}.$$
$$\frac{x^2}{x^2-a^2}=\frac{x^2}{(x+a)(x-a)}.$$

2. $\dfrac{a}{a+b},\ \dfrac{a^2}{a^2-b^2}$.

$$\frac{a}{a+b}=\frac{a}{a+b}$$
$$\frac{a^2}{a^2-b^2}=\frac{a^2}{(a+b)(a-b)}.$$
$\therefore$ the L. C. D. $=(a+b)(a-b)$
$$\frac{a}{a+b}=\frac{a(a-b)}{(a+b)(a-b)}.$$
$$\frac{a^2}{a^2-b^2}=\frac{a^2}{(a+b)(a-b)}.$$

3. $\dfrac{1}{1+2a},\ \dfrac{1}{1-4a^2}$.

$$\frac{1}{1+2a}=\frac{1}{1+2a}$$
$$\frac{1}{1-4a^2}=\frac{1}{(1+2a)(1-2a)}.$$
$\therefore$ the L. C. D. $=(1+2a)(1-2a)$
$$\frac{1}{1+2a}=\frac{1-2a}{(1+2a)(1-2a)}.$$
$$\frac{1}{1-4a^2}=\frac{1}{(1+2a)(1-2a)}.$$

4. $\frac{9}{16-x^2}$, $\frac{4-x}{4+x}$.

$$\frac{9}{16-x^2}=\frac{9}{(4+x)(4-x)}$$
$$\frac{4-x}{4+x}=\frac{4-x}{4+x}.$$
$$\therefore \text{ the L. C. D.} = (4+x)(4-x)$$
$$\frac{9}{16-x^2}=\frac{9}{(4+x)(4-x)}.$$
$$\frac{4-x}{4+x}=\frac{(4-x)^2}{(4+x)(4-x)}.$$

5. $\frac{a^2}{27-a^3}$, $\frac{a}{3-a}$.

$$\frac{a^2}{27-a^3}=\frac{a^2}{(3-a)(9+3a+a^2)}$$
$$\frac{a}{3-a}=\frac{a}{3-a}.$$
$$\therefore \text{ the L. C. D.} = (3-a)(9+3a+a^2)$$
$$\frac{a^2}{27-a^3}=\frac{a^2}{(3-a)(9+3a+a^2)}.$$
$$\frac{a}{3-a}=\frac{a(9+3a+a^2)}{(3-a)(9+3a+a^2)}.$$

6. $\frac{1}{x^2-5x+6}$, $\frac{1}{x^2-x-6}$.

$$\frac{1}{x^2-5x+6}=\frac{1}{(x-2)(x-3)}$$
$$\frac{1}{x^2-x-6}=\frac{1}{(x+2)(x-3)}.$$
$$\therefore \text{ the L. C. D.} = (x+2)(x-2)(x-3)$$
$$\frac{1}{x^2-5x+6}=\frac{x+2}{(x+2)(x-2)(x-3)}.$$
$$\frac{1}{x^2-x-6}=\frac{x-2}{(x+2)(x-2)(x-3)}.$$

Exercise 46. Page 95.

Find the sum of:

1. $\frac{x+1}{2}+\frac{x-3}{5}+\frac{x+5}{10}.$

The L. C. D. = 10.
The multipliers are 5, 2, and 1 respectively.

$$\begin{aligned}5(x+1) &= 5x+5 = \text{1st numerator.}\\ 2(x-3) &= 2x-6 = \text{2d numerator.}\\ 1(x+5) &= \underline{x+5} = \text{3d numerator.}\\ &\ 8x+4\\ \text{or}\quad 2(4x+2) &= \text{sum of numerators.}\end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{2(4x+2)}{10} = \frac{4x+2}{5}.$$

2. $\frac{2x-1}{3}+\frac{x+5}{4}+\frac{x-4}{6}.$

The L. C. D. = 12.
The multipliers are 4, 3, and 2 respectively.

$$\begin{aligned}4(2x-1) &= 8x-\ \ 4 = \text{1st numerator.}\\ 3(x+5) &= 3x+15 = \text{2d numerator.}\\ 2(x-4) &= \underline{2x-\ \ 8} = \text{3d numerator.}\\ &\ 13x+\ \ 3 = \text{sum of numerators.}\end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{13x+3}{12}.$$

3. $\frac{7x-1}{6}-\frac{3x-2}{7}+\frac{x-5}{3}.$

The L. C. D. = 42.
The multipliers are 7, 6, and 14 respectively.

$$\begin{aligned}7(7x-1) &= \quad 49x-\ 7 = \text{1st numerator.}\\ -6(3x-2) &= -18x+12 = \text{2d numerator.}\\ 14(x-5) &= \quad \underline{14x-70} = \text{3d numerator.}\\ &\quad 45x-65\\ \text{or}\quad 5(9x-13) &= \text{sum of numerators.}\end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{5(9x-13)}{42}.$$

4. $\dfrac{3x-2}{9} - \dfrac{x-2}{6} + \dfrac{5x+3}{4}$.

The L. C. D. = 36.

The multipliers are 4, 6, and 9 respectively.

$$\begin{aligned} 4(3x-2) &= \quad 12x - \;\; 8 = \text{1st numerator.} \\ -6(x-2) &= -\;\; 6x + 12 = \text{2d numerator.} \\ 9(5x+3) &= \quad \underline{45x + 27} = \text{3d numerator.} \\ & \qquad 51x + 31 = \text{sum of numerators.} \end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{51x+31}{36}.$$

5. $\dfrac{x-1}{6} - \dfrac{x-3}{3} + \dfrac{x-5}{2}$.

The L. C. D. = 6.

The multipliers are 1, 2, and 3 respectively.

$$\begin{aligned} 1(x-1) &= \qquad x - \;\; 1 = \text{1st numerator.} \\ -2(x-3) &= -2x + \;\; 6 = \text{2d numerator.} \\ 3(x-5) &= \quad \underline{3x - 15} = \text{3d numerator.} \\ & \qquad 2x - 10 \\ \text{or} \quad & \quad 2(x-5) = \text{sum of numerators.} \end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{2(x-5)}{6} = \frac{x-5}{3}.$$

6. $\dfrac{x-2y}{2x} + \dfrac{x+5y}{4x} - \dfrac{x+7y}{8x}$.

The L. C. D. = $8x$.

The multipliers are 4, 2, and 1 respectively.

$$\begin{aligned} 4(x-2y) &= \quad 4x - \;\; 8y = \text{1st numerator.} \\ 2(x+5y) &= \quad 2x + 10y = \text{2d numerator.} \\ -1(x+7y) &= -\underline{\;\; x - \;\; 7y} = \text{3d numerator.} \\ & \qquad 5x - 5y \\ \text{or} \quad & \quad 5(x-y) = \text{sum of numerators.} \end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{5(x-y)}{8x}.$$

7. $\dfrac{5x-11}{3} - \dfrac{2x-1}{10} - \dfrac{11x-5}{15}$.

The L. C. D. = 30.

The multipliers are 10, 3, and 2 respectively.

$$\begin{aligned}
10(5x-11) &= 50x-110 = \text{1st numerator.}\\
-3(2x-1) &= -6x+3 = \text{2d numerator.}\\
-2(11x-5) &= \underline{-22x+10} = \text{3d numerator.}\\
&22x-97 = \text{sum of numerators.}
\end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{22x-97}{30}.$$

8. $\dfrac{x-3}{3x} - \dfrac{x^2-6x}{5x^2} - \dfrac{7x^2-x^3}{15x^3}.$

The L. C. D. $= 15x^3$.

The multipliers are $5x^2$, $3x$, and 1 respectively.

$$\begin{aligned}
5x^2(x-3) &= 5x^3-15x^2 = \text{1st numerator.}\\
-3x(x^2-6x) &= -3x^3+18x^2 = \text{2d numerator.}\\
-1(7x^2-x^3) &= \underline{x^3-7x^2} = \text{3d numerator.}\\
&3x^3-4x^2\\
\text{or}\quad &x^2(3x-4) = \text{sum of numerators.}
\end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{x^2(3x-4)}{15x^3} = \frac{3x-4}{15x}.$$

9. $\dfrac{ac-b^2}{ac} - \dfrac{ab-c^2}{ab} + \dfrac{a^2-bc}{bc}.$

The L. C. D. $= abc$.

The multipliers are b, c, and a respectively.

$$\begin{aligned}
b(ac-b^2) &= abc \qquad -b^3 \qquad\quad = \text{1st numerator.}\\
-c(ab-c^2) &= -abc \qquad\qquad\quad +c^3 = \text{2d numerator.}\\
a(a^2-bc) &= \underline{-abc+a^3 \qquad\qquad\quad} = \text{3d numerator.}\\
&-abc+a^3-b^3+c^3 = \text{sum of numerators.}
\end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{a^3-b^3+c^3-abc}{abc}.$$

Exercise 47. Page 96.

Find the sum of :

1. $\dfrac{1}{x+3} + \dfrac{1}{x-2}.$

The L. C. D. $= (x+3)(x-2)$.

The multipliers are $x-2$ and $x+3$ respectively.

$$\begin{aligned}
1(x-2) &= x-2 = \text{1st numerator.}\\
1(x+3) &= \underline{x+3} = \text{2d numerator.}\\
&2x+1 = \text{sum of numerators.}
\end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{2x+1}{(x+3)(x-2)}.$$

2. $\frac{1}{x+1}+\frac{1}{x-1}$.

The L. C. D. $=(x+1)(x-1)$.
The multipliers are $x-1$ and $x+1$ respectively.

$$\begin{aligned} 1(x-1) &= x-1 = \text{1st numerator.} \\ 1(x+1) &= \underline{x+1} = \text{2d numerator.} \\ & \quad 2x \quad = \text{sum of numerators.} \end{aligned}$$

$\therefore$ sum of fractions $=\frac{2x}{x^2-1}$.

3. $\frac{4}{x-8}-\frac{1}{x+2}$.

The L. C. D. $=(x-8)(x+2)$.
The multipliers are $x+2$ and $x-8$ respectively.

$$\begin{aligned} 4(x+2) &= 4x+8 = \text{1st numerator.} \\ -1(x-8) &= \underline{-x+8} = \text{2d numerator.} \\ & \quad 3x+16 = \text{sum of numerators.} \end{aligned}$$

$\therefore$ sum of fractions $=\frac{3x+16}{(x-8)(x+2)}$.

4. $\frac{a+x}{a-x}-\frac{a-x}{a+x}$.

The L. C. D. $=(a-x)(a+x)$.
The multipliers are $a+x$ and $a-x$ respectively.

$$\begin{aligned} (a+x)(a+x) &= a^2+2ax+x^2 = \text{1st numerator.} \\ -(a-x)(a-x) &= \underline{-a^2+2ax-x^2} = \text{2d numerator.} \\ & \quad 4ax \quad = \text{sum of numerators.} \end{aligned}$$

$\therefore$ sum of fractions $=\frac{4ax}{a^2-x^2}$.

5. $\frac{x}{x-a}-\frac{x^2}{x^2-a^2}$.

The L. C. D. $=(x+a)(x-a)$.
The multipliers are $x+a$ and 1 respectively.

$$\begin{aligned} x(x+a) &= x^2+ax = \text{1st numerator.} \\ -1(x^2) &= \underline{-x^2 \quad\quad} = \text{2d numerator.} \\ & \quad\quad ax = \text{sum of numerators.} \end{aligned}$$

$\therefore$ sum of fractions $=\frac{ax}{x^2-a^2}$.

6. $\frac{4a^2+b^2}{4a^2-b^2}-\frac{2a+b}{2a-b}$.

The L. C. D. $=(2a+b)(2a-b)$.
The multipliers are 1 and $2a+b$ respectively.

$$\begin{array}{rcll}1(4a^2+b^2) &=& 4a^2 \qquad\quad + b^2 &= \text{1st numerator.}\\ -(2a+b)(2a+b) &=& \underline{-4a^2-4ab-b^2} &= \text{2d numerator.}\\ && -4ab &= \text{sum of numerators.}\end{array}$$

$$\therefore \text{sum of fractions} = -\frac{4ab}{4a^2-b^2}.$$

7. $\frac{7}{9-a^2}-\frac{1}{3+a}-\frac{1}{3-a}$.

The L. C. D. $=(3+a)(3-a)$.
The multipliers are 1, $3-a$, and $3+a$ respectively.

$$\begin{array}{rcll}1(7) &=& 7 &= \text{1st numerator.}\\ -(3-a) &=& -3+a &= \text{2d numerator.}\\ -(3+a) &=& \underline{-3-a} &= \text{3d numerator.}\\ && 1 &= \text{sum of numerators.}\end{array}$$

$$\therefore \text{sum of fractions} = \frac{1}{9-a^2}.$$

8. $\frac{1}{a-b}-\frac{1}{a+b}-\frac{b}{a^2-b^2}$.

The L. C. D. $=(a+b)(a-b)$.
The multipliers are $a+b$, $a-b$, and 1 respectively.

$$\begin{array}{rcll}1(a+b) &=& a+b &= \text{1st numerator.}\\ -1(a-b) &=& -a+b &= \text{2d numerator.}\\ -1(b) &=& \underline{\qquad -b} &= \text{3d numerator.}\\ && b &= \text{sum of numerators.}\end{array}$$

$$\therefore \text{sum of fractions} = \frac{b}{a^2-b^2}.$$

9. $\frac{2}{x-2}-\frac{2}{x+2}+\frac{5x}{x^2-4}$.

The L. C. D. $=(x+2)(x-2)$.
The multipliers are $x+2$, $x-2$, and 1 respectively.

$$\begin{array}{rcll}2(x+2) &=& 2x+4 &= \text{1st numerator.}\\ -2(x-2) &=& -2x+4 &= \text{2d numerator.}\\ 1(5x) &=& \underline{5x \qquad} &= \text{3d numerator.}\\ && 5x+8 &= \text{sum of numerators.}\end{array}$$

$$\therefore \text{sum of fractions} = \frac{5x+8}{x^2-4}.$$

10. $\dfrac{3-x}{1-3x}-\dfrac{3+x}{1+3x}-\dfrac{15x-1}{1-9x^2}.$

The L. C. D. $=(1+3x)(1-3x)$.

The multipliers are $1+3x$, $1-3x$, and 1 respectively.

$$\begin{array}{rcll} (1+3x)(3-x) &=& 3+8x-3x^2 &= \text{1st numerator.}\\ -(1-3x)(3+x) &=& -3+8x+3x^2 &= \text{2d numerator.}\\ -1(15x-1) &=& 1-15x &= \text{3d numerator.}\\ \hline && 1+x &= \text{sum of numerators.} \end{array}$$

$$\therefore \text{sum of fractions} = \frac{1+x}{1-9x^2}.$$

11. $\dfrac{1}{a}-\dfrac{1}{a+3}+\dfrac{3}{a+1}.$

The L. C. D. $=a(a+1)(a+3)$.

The multipliers are $(a+1)(a+3)$, $a(a+1)$, and $a(a+3)$ respectively.

$$\begin{array}{rcll} 1(a+1)(a+3) &=& a^2+4a+3 &= \text{1st numerator.}\\ -a(a+1) &=& -a^2-a &= \text{2d numerator.}\\ 3a(a+3) &=& 3a^2+9a &= \text{3d numerator.}\\ \hline && 3a^2+12a+3 & \\ && \text{or } 3(a^2+4a+1) &= \text{sum of numerators.} \end{array}$$

$$\therefore \text{sum of fractions} = \frac{3(a^2+4a+1)}{a(a+1)(a+3)}.$$

12. $\dfrac{x}{x-1}-1-\dfrac{1}{x+1}.$

The L. C. D. $=(x+1)(x-1)$.

The multipliers are $x+1$, $(x+1)(x-1)$, and $x-1$ respectively.

$$\begin{array}{rcll} x(x+1) &=& x^2+x &= \text{1st numerator.}\\ -1(x+1)(x-1) &=& -x^2+1 &= \text{2d numerator.}\\ -1(x-1) &=& -x+1 &= \text{3d numerator.}\\ \hline && 2 &= \text{sum of numerators.} \end{array}$$

$$\therefore \text{sum of fractions} = \frac{2}{x^2-1}.$$

13. $\dfrac{x+1}{x+2}+\dfrac{x-2}{x-3}+\dfrac{2x+7}{x^2-x-6}.$

The L. C. D. $=(x+2)(x-3)$.

The multipliers are $x-3$, $x+2$, and 1 respectively.

$$\begin{aligned}(x+1)(x-3) &= x^2-2x-3 = \text{1st numerator.}\\ (x-2)(x+2) &= x^2 \qquad\quad -4 = \text{2d numerator.}\\ 1(2x+7) &= \underline{\qquad\quad 2x+7} = \text{3d numerator.}\\ & \quad 2x^2 \qquad\qquad = \text{sum of numerators.}\end{aligned}$$

$$\therefore \text{ sum of fractions} = \frac{2x^2}{(x+2)(x-3)}.$$

14. $\dfrac{1}{x(x-1)} - \dfrac{2}{x^2-1} + \dfrac{1}{x(x+1)}.$

The L. C. D. $= x(x+1)(x-1)$.

The multipliers are $x+1$, x, and $x-1$ respectively.

$$\begin{aligned}1(x+1) &= \quad x+1 = \text{1st numerator.}\\ -2(x) &= -2x \qquad = \text{2d numerator.}\\ 1(x-1) &= \underline{\quad x-1} = \text{3d numerator.}\\ & \quad 0 \qquad\quad = \text{sum of numerators.}\end{aligned}$$

$\therefore$ sum of fractions $= 0$.

Exercise 48. Page 97.

Find the sum of:

1. $\dfrac{1}{2x+1} + \dfrac{1}{2x-1} - \dfrac{4x}{4x^2-1}.$

The L. C. D. $= (2x+1)(2x-1)$.

The multipliers are $2x-1$, $2x+1$, and 1 respectively.

$$\begin{aligned}1(2x-1) &= \quad 2x-1 = \text{1st numerator.}\\ 1(2x+1) &= \quad 2x+1 = \text{2d numerator.}\\ -1(4x) &= \underline{-4x \qquad} = \text{3d numerator.}\\ & \quad 0 \qquad\quad = \text{sum of numerators.}\end{aligned}$$

$\therefore$ sum of fractions $= 0$.

2. $\dfrac{a^2+b^2}{a^2-b^2} + \dfrac{a}{a+b} - \dfrac{b}{a-b}.$

The L. C. D. $= (a+b)(a-b)$.

The multipliers are 1, $a-b$, and $a+b$ respectively.

$$\begin{aligned}1(a^2+b^2) &= a^2 \qquad\quad +b^2 = \text{1st numerator.}\\ a(a-b) &= a^2- \ ab \qquad = \text{2d numerator.}\\ -b(a+b) &= \underline{\qquad - \ ab-b^2} = \text{3d numerator.}\\ & \quad 2a^2-2ab\\ & \text{or } \ 2a(a-b) = \text{sum of numerators.}\end{aligned}$$

$$\therefore \text{ sum of fractions} = \frac{2a(a-b)}{(a+b)(a-b)} = \frac{2a}{a+b}.$$

3. $\frac{3a}{1-a^2}+\frac{2}{1-a}-\frac{2}{1+a}$.

The L. C. D. $=(1-a)(1+a)$.
The multipliers are 1, $1+a$, and $1-a$ respectively.

$$\begin{aligned}
1(3a) &= 3a &&= \text{1st numerator.}\\
2(1+a) &= 2a+2 &&= \text{2d numerator.}\\
-2(1-a) &= \underline{2a-2} &&= \text{3d numerator.}\\
& 7a &&= \text{sum of numerators.}
\end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{7a}{1-a^2}.$$

4. $\frac{1}{2x+5y}-\frac{3x}{4x^2-25y^2}+\frac{1}{2x+5y}$.

The L. C. D. $=(2x+5y)(2x-5y)$.
The multipliers are $2x-5y$, 1, and $2x-5y$ respectively.

$$\begin{aligned}
1(2x-5y) &= 2x-5y &&= \text{1st numerator.}\\
-1(3x) &= -3x &&= \text{2d numerator.}\\
1(2x-5y) &= \underline{2x-5y} &&= \text{3d numerator.}\\
& x-10y &&= \text{sum of numerators.}
\end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{x-10y}{4x^2-25y^2}.$$

5. $\frac{1}{x+4y}-\frac{8y}{x^2-16y^2}+\frac{1}{x-4y}$.

The L. C. D. $=(x+4y)(x-4y)$.
The multipliers are $x-4y$, 1, and $x+4y$ respectively.

$$\begin{aligned}
1(x-4y) &= x-4y &&= \text{1st numerator.}\\
-1(8y) &= -8y &&= \text{2d numerator.}\\
1(x+4y) &= \underline{x+4y} &&= \text{3d numerator.}\\
& 2x-8y &&\\
\text{or } & 2(x-4y) &&= \text{sum of numerators.}
\end{aligned}$$

$$\therefore \text{sum of fractions} = \frac{2(x-4y)}{(x+4y)(x-4y)} = \frac{2}{x+4y}.$$

6. $\frac{3}{2x-3}-\frac{2}{3+2x}-\frac{3}{4x^2-9}$.

The L. C. D. $=(2x+3)(2x-3)$.
The multipliers are $2x+3$, $2x-3$, and 1 respectively.

$$\begin{array}{rcrl}
3\,(2\,x+3) & = & 6\,x+\ \ 9 & = \text{1st numerator.} \\
-\,2\,(2\,x-3) & = & -\,4\,x+\ \ 6 & = \text{2d numerator.} \\
-\,3 & = & -\ \ 3 & = \text{3d numerator.} \\
\hline
 & & 2\,x+12 & \\
 & & \text{or } 2\,(x+6) & = \text{sum of numerators.}
\end{array}$$

$$\therefore \text{ sum of fractions } = \frac{2\,(x+6)}{4\,x^2-9}.$$

Exercise 49. Page 100.

Express in the simplest form:

1. $\dfrac{15\,a^2}{7\,b^2}\times\dfrac{28\,ab}{9\,a^3c}.$

$$\frac{15\,a^2}{7\,b^2}\times\frac{28\,ab}{9\,a^3c}=\frac{15\times 28\times a^3b}{7\times 9\,a^3b^2c}$$
$$=\frac{20}{3\,bc}.$$

2. $\dfrac{3\,x^2y^2z^3}{4\,a^2b^2c^2}\times\dfrac{8\,a^3b^2c^2}{9\,x^2yz^3}.$

$$\frac{3\,x^2y^2z^3}{4\,a^2b^2c^2}\times\frac{8\,a^3b^2c^2}{9\,x^2yz^3}=\frac{3\times 8\,a^3b^2c^2x^2y^2z^3}{4\times 9\,a^2b^2c^2x^2yz^3}$$
$$=\frac{2\,ay}{3}.$$

3. $\dfrac{5\,m^2n^2p^4}{3\,x^2yz^3}\times\dfrac{21\,xyz^2}{20\,m^2n^2p^2}.$

$$\frac{5\,m^2n^2p^4}{3\,x^2yz^3}\times\frac{24\,xyz^2}{20\,m^2n^2p^2}=\frac{5\times 21\,m^2n^2p^4xyz^2}{3\times 20\,m^2n^2p^2x^2yz^3}$$
$$=\frac{7\,p^2}{4\,xz}.$$

4. $\dfrac{16\,a^2b^2c^3}{21\,m^2x^3y^4}\times\dfrac{3\,m^3x^3y^4}{8\,a^2b^2c^2}.$

$$\frac{16\,a^4b^2c^3}{21\,m^2x^3y^4}\times\frac{3\,m^3x^3y^4}{8\,a^2b^2c^2}=\frac{16\times 3\,a^4b^2c^3m^3x^3y^4}{21\times 8\,a^2b^2c^2m^2x^3y^4}$$
$$=\frac{2\,a^2cm}{7}.$$

5. $\frac{2a}{bc} \times \frac{3b}{ac} \times \frac{5c}{ab}$.

$$\frac{2a}{bc} \times \frac{3b}{ac} \times \frac{5c}{ab} = \frac{2 \times 3 \times 5\,abc}{a^2b^2c^2}$$
$$= \frac{30}{abc}.$$

6. $\frac{2a^3}{3bc} \times \frac{3b^3}{5ac} \times \frac{5c^3}{2ab}$.

$$\frac{2a^3}{3bc} \times \frac{3b^3}{5ac} \times \frac{5c^3}{2ab} = \frac{2 \times 3 \times 5\,a^3b^3c^3}{3 \times 5 \times 2\,a^2b^2c^2}$$
$$= abc.$$

7. $\frac{5abc^3}{3x^2} \div \frac{10ac^3}{6bx^2}$.

$$\frac{5abc^3}{3x^2} \div \frac{10ac^3}{6bx^2} = \frac{5abc^3}{3x^2} \times \frac{6bx^2}{10ac^3}$$
$$= \frac{5 \times 6\,ab^2c^3x^2}{3 \times 10\,ac^3x^2}$$
$$= b^2.$$

8. $\frac{x^2 - a^2}{x^2 - 4a^2} \times \frac{x + 2a}{x - a}$.

$$\frac{x^2 - a^2}{x^2 - 4a^2} \times \frac{x + 2a}{x - a} = \frac{(x + a)(x - a)(x + 2a)}{(x + 2a)(x - 2a)(x - a)}$$
$$= \frac{x + a}{x - 2a}.$$

9. $\frac{x^2y^2 + 3xy}{4c^2 - 1} \times \frac{2c + 1}{xy + 3}$.

$$\frac{x^2y^2 + 3xy}{4c^2 - 1} \times \frac{2c + 1}{xy + 3} = \frac{xy(xy + 3)(2c + 1)}{(2c + 1)(2c - 1)(xy + 3)}$$
$$= \frac{xy}{2c - 1}.$$

10. $\frac{a^2 - 100}{a^2 - 9} \times \frac{a - 3}{a - 10}$.

$$\frac{a^2 - 100}{a^2 - 9} \times \frac{a - 3}{a - 10} = \frac{(a + 10)(a - 10)(a - 3)}{(a + 3)(a - 3)(a - 10)}$$
$$= \frac{a + 10}{a + 3}.$$

11. $\dfrac{9x^2-4y^2}{x^2-4}\times\dfrac{x+2}{3x-2y}.$

$$\frac{9x^2-4y^2}{x^2-4}\times\frac{x+2}{3x-2y}=\frac{(3x+2y)(3x-2y)(x+2)}{(x+2)(x-2)(3x-2y)}$$
$$=\frac{3x+2y}{x-2}.$$

12. $\dfrac{25a^2-b^2}{16a^2-9b^2}\div\dfrac{5a-b}{4a-3b}.$

$$\frac{25a^2-b^2}{16a^2-9b^2}\div\frac{5a-b}{4a-3b}=\frac{(5a+b)(5a-b)(4a-3b)}{(4a+3b)(4a-3b)(5a-b)}$$
$$=\frac{5a+b}{4a+3b}.$$

13. $\dfrac{x^2-49}{(a+b)^2-c^2}\div\dfrac{x+7}{(a+b)-c}.$

$$\frac{x^2-49}{(a+b)^2-c^2}\div\frac{x+7}{(a+b)-c}=\frac{(x+7)(x-7)(a+b-c)}{(a+b-c)(a+b+c)(x+7)}$$
$$=\frac{x-7}{a+b+c}.$$

14. $\dfrac{x^2+2x+1}{x^2-25}\div\dfrac{x+1}{x^2+5x}.$

$$\frac{x^2+2x+1}{x^2-25}\div\frac{x+1}{x^2+5x}=\frac{(x+1)(x+1)x(x+5)}{(x+5)(x-5)(x+1)}$$
$$=\frac{x(x+1)}{x-5}.$$

15. $\dfrac{a^2+3a+2}{a^2+5a+6}\times\dfrac{a^2+7a+12}{a^2+9a+20}.$

$$\frac{a^2+3a+2}{a^2+5a+6}\times\frac{a^2+7a+12}{a^2+9a+20}=\frac{(a+1)(a+2)(a+3)(a+4)}{(a+2)(a+3)(a+4)(a+5)}$$
$$=\frac{a+1}{a+5}.$$

16. $\dfrac{y^2-y-30}{y^2-36}\times\dfrac{y^2-y-2}{y^2+3y-10}\times\dfrac{y^2+6y}{y^2+y}.$

$$\frac{y^2-y-30}{y^2-36}\times\frac{y^2-y-2}{y^2+3y-10}\times\frac{y^2+6y}{y^2+y}.$$
$$=\frac{(y+5)(y-6)(y+1)(y-2)y(y+6)}{(y+6)(y-6)(y-2)(y+5)y(y+1)}$$
$$=1.$$

17. $\dfrac{x^2-2x+1}{x^2-y^2}\times\dfrac{x^2+2xy+y^2}{x-1}\div\dfrac{x^2-1}{x^2-xy}$.

$$\begin{aligned}\frac{x^2-2x+1}{x^2-y^2}\times\frac{x^2+2xy+y^2}{x-1}\div\frac{x^2-1}{x^2-xy}\\ &=\frac{(x-1)(x-1)}{(x+y)(x-y)}\times\frac{(x+y)(x+y)}{x-1}\times\frac{x(x-y)}{(x+1)(x-1)}\\ &=\frac{x(x+y)}{x+1}.\end{aligned}$$

18. $\dfrac{a^2-b^2}{a^2-3ab+2b^2}\times\dfrac{ab-2b^2}{a^2+ab}\div\dfrac{(a-b)^2}{a(a-b)}$.

$$\begin{aligned}\frac{a^2-b^2}{a^2-3ab+2b^2}\times\frac{ab-2b^2}{a^2+ab}\div\frac{(a-b)^2}{a(a-b)}\\ &=\frac{(a+b)(a-b)}{(a-2b)(a-b)}\times\frac{b(a-2b)}{a(a+b)}\times\frac{a(a-b)}{(a-b)(a-b)}\\ &=\frac{b}{a-b}.\end{aligned}$$

19. $\dfrac{(a+b)^2-c^2}{a^2+ab-ac}\times\dfrac{a^2b^2c^2}{a^2+ab+ac}\div\dfrac{b^2c^2}{abc}$.

$$\begin{aligned}\frac{(a+b)^2-c^2}{a^2+ab-ac}\times\frac{a^2b^2c^2}{a^2+ab+ac}\div\frac{b^2c^2}{abc}\\ &=\frac{(a+b+c)(a+b-c)}{a(a+b-c)}\times\frac{a^2b^2c^2}{a(a+b+c)}\times\frac{abc}{b^2c^2}\\ &=abc.\end{aligned}$$

20. $\dfrac{x^2+7xy+10y^2}{x^2+6xy+5y^2}\times\dfrac{x+1}{x^2+4x+4}\div\dfrac{1}{x+2}$.

$$\begin{aligned}\frac{x^2+7xy+10y^2}{x^2+6xy+5y^2}\times\frac{x+1}{x^2+4x+4}\div\frac{1}{x+2}\\ &=\frac{(x+2y)(x+5y)}{(x+y)(x+5y)}\times\frac{x+1}{(x+2)(x+2)}\times\frac{x+2}{1}\\ &=\frac{(x+2y)(x+1)}{(x+y)(x+2)}.\end{aligned}$$

Exercise 50. Page 102.

Reduce to the simplest form:

1. $\dfrac{\dfrac{x}{b}+\dfrac{y}{b}}{\dfrac{z}{b}}.$

Multiply both terms by b.

$$\frac{\dfrac{x}{b}+\dfrac{y}{b}}{\dfrac{z}{b}}=\frac{x+y}{z}.$$

2. $\dfrac{x+\dfrac{y}{4}}{x-\dfrac{y}{3}}.$

Multiply both terms by 12.

$$\frac{x+\dfrac{y}{4}}{x-\dfrac{y}{3}}=\frac{12x+3y}{12x-4y}.$$

3. $\dfrac{\dfrac{ab}{7}-3d}{3c-\dfrac{ab}{d}}.$

Multiply both terms by $7d$.

$$\frac{\dfrac{ab}{7}-3d}{3c-\dfrac{ab}{d}}=\frac{abd-21d^2}{21cd-7ab}.$$

4. $\dfrac{1+\dfrac{1}{x+1}}{1-\dfrac{1}{x-1}}.$

Multiply both terms by x^2-1.

$$\frac{1+\dfrac{1}{x+1}}{1-\dfrac{1}{x-1}}=\frac{x^2-1+x-1}{x^2-1-(x+1)}=\frac{x^2+x-2}{x^2-x-2}.$$

5. $\dfrac{\dfrac{2m+x}{m+x}-1}{1-\dfrac{x}{m+x}}.$

Multiply both terms by $m+x$.

$$\frac{\dfrac{2m+x}{m+x}-1}{1-\dfrac{x}{m+x}}=\frac{2m+x-m-x}{m+x-x}=\frac{m}{m}=1.$$

6. $\dfrac{\dfrac{x+y}{x^2-y^2}}{\dfrac{x-y}{x+y}}$.

Multiply both terms by x^2-y^2.

$$\frac{\dfrac{x+y}{x^2-y^2}}{\dfrac{x-y}{x+y}}=\frac{x+y}{x^2-2xy+y^2}.$$

7. $\dfrac{a+\dfrac{ab}{a-b}}{a-\dfrac{ab}{a+b}}$.

Multiply both terms by a^2-b^2.

$$\frac{a+\dfrac{ab}{a-b}}{a-\dfrac{ab}{a+b}}=\frac{a^3-ab^2+a^2b+ab^2}{a^3-ab^2-a^2b+ab^2}$$
$$=\frac{a^3+a^2b}{a^3-a^2b}=\frac{a^2(a+b)}{a^2(a-b)}$$
$$=\frac{a+b}{a-b}.$$

8. $\dfrac{9a^2-64}{a-1-\dfrac{a+4}{4}}$.

Multiply both terms by 4.

$$\frac{9a^2-64}{a-1-\dfrac{a+4}{4}}=\frac{4(9a^2-64)}{4a-4-a-4}$$
$$=\frac{4(3a+8)(3a-8)}{3a-8}$$
$$=4(3a+8).$$

9. $\dfrac{\dfrac{1}{x}+\dfrac{1}{y}}{\dfrac{1}{x}-\dfrac{1}{y}}$.

Multiply both terms by xy.

$$\frac{\dfrac{1}{x}+\dfrac{1}{y}}{\dfrac{1}{x}-\dfrac{1}{y}}=\frac{y+x}{y-x}.$$

10. $\dfrac{x+3+\dfrac{2}{x}}{1+\dfrac{3}{x}+\dfrac{2}{x^2}}$.

Multiply both terms by x^2.

$$\frac{x+3+\dfrac{2}{x}}{1+\dfrac{3}{x}+\dfrac{2}{x^2}}=\frac{x^3+3x^2+2x}{x^2+3x+2}=\frac{x(x^2+3x+2)}{x^2+3x+2}=x.$$

11. $\dfrac{\dfrac{1}{x}-\dfrac{2}{x^2}+\dfrac{1}{x^3}}{\dfrac{(1-x)^2}{x^2}}$.

Multiply both terms by x^3.

$$\frac{\dfrac{1}{x}-\dfrac{2}{x^2}+\dfrac{1}{x^3}}{\dfrac{(1-x)^2}{x^2}}=\frac{x^2-2x+1}{x(1-x)^2}=\frac{1-2x+x^2}{x(1-x)^2}=\frac{(1-x)^2}{x(1-x)^2}=\frac{1}{x}.$$

12. $\dfrac{x^2-x-6}{1-\dfrac{4}{x^2}}$.

Multiply both terms by x^2.

$$\frac{x^2-x-6}{1-\dfrac{4}{x^2}}=\frac{x^2(x^2-x-6)}{x^2-4}=\frac{x^2(x+2)(x-3)}{(x+2)(x-2)}=\frac{x^2(x-3)}{x-2}.$$

13. $\dfrac{a^2-a+\dfrac{a-1}{a+1}}{a+\dfrac{1}{a+1}}$.

Multiply both terms by $a+1$.

$$\frac{a^2-a+\dfrac{a-1}{a+1}}{a+\dfrac{1}{a+1}}=\frac{a^3-a+a-1}{a^2+a+1}=\frac{a^3-1}{a^2+a+1}$$

$$=\frac{(a-1)(a^2+a+1)}{a^2+a+1}=a-1.$$

14. $\dfrac{\dfrac{4a(a-x)}{a^2-x^2}}{\dfrac{a-x}{a+x}}$.

Multiply both terms by a^2-x^2.

$$\frac{\dfrac{4a(a-x)}{a^2-x^2}}{\dfrac{a-x}{a+x}}=\frac{4a(a-x)}{(a-x)(a-x)}=\frac{4a}{a-x}.$$

Exercise 51. Page 105.

Solve:

1. $\dfrac{x-1}{2}=\dfrac{x+1}{3}$.

$$\frac{x-1}{2}=\frac{x+1}{3}.$$

Multiply by the L. C. D., 6.

$$3x-3=2x+2$$
$$3x-2x=2+3.$$
$$\therefore x=5.$$

2. $\dfrac{3x-1}{4}=\dfrac{2x+1}{3}$.

$$\frac{3x-1}{4}=\frac{2x+1}{3}.$$

Multiply by the L. C. D., 12.

$$9x-3=8x+4$$
$$9x-8x=4+3.$$
$$\therefore x=7.$$

3. $\frac{6x-19}{2}=\frac{2x-11}{3}.$

$$\frac{6x-19}{2}=\frac{2x-11}{3}.$$

Multiply by the L. C. D., 6.

$$18x-57=4x-22$$
$$18x-4x=57-22$$
$$14x=35$$
$$2x=5.$$
$$\therefore x=2\tfrac{1}{2}.$$

4. $\frac{7x-40}{8}=\frac{9x-80}{10}.$

$$\frac{7x-40}{8}=\frac{9x-80}{10}.$$

Multiply by the L. C. D., 40.

$$35x-200=36x-320$$
$$35x-36x=-320+200$$
$$-x=-120.$$
$$\therefore x=120.$$

5. $\frac{3x-116}{4}+\frac{180-5x}{6}=0.$

$$\frac{3x-116}{4}+\frac{180-5x}{6}=0.$$

Multiply by the L. C. D., 12.

$$9x-348+360-10x=0$$
$$9x-10x=348-360$$
$$-x=-12.$$
$$\therefore x=12.$$

6. $\frac{3x-4}{2}-\frac{3x-1}{16}=\frac{6x-5}{8}.$

$$\frac{3x-4}{2}-\frac{3x-1}{16}=\frac{6x-5}{8}.$$

Multiply by the L. C. D., 16.

$$24x-32-3x+1=12x-10$$
$$24x-3x-12x=32-1-10$$
$$9x=21.$$
$$\therefore x=2\tfrac{1}{3}.$$

7. $\frac{x-1}{8}-\frac{x+1}{18}=1.$

$$\frac{x-1}{8}-\frac{x+1}{18}=1.$$

Multiply by the L. C. D., 72.

$$9x-9-4x-4=72$$
$$9x-4x=72+9+4$$
$$5x=85.$$
$$\therefore x=17.$$

8. $\frac{60-x}{14}-\frac{3x-5}{7}=\frac{3x}{4}.$

$$\frac{60-x}{14}-\frac{3x-5}{7}=\frac{3x}{4}.$$

Multiply by the L. C. D., 28.

$$120-2x-12x+20=21x$$
$$-2x-12x-21x=-120-20$$
$$-35x=-140.$$
$$\therefore x=4.$$

9. $\frac{3x-1}{11}-\frac{2-x}{10}=\frac{6}{5}.$

$$\frac{3x-1}{11}-\frac{2-x}{10}=\frac{6}{5}.$$

Multiply by the L. C. D., 110.

$$30x-10-22+11x=132$$
$$30x+11x=132+10+22$$
$$41x=164.$$
$$\therefore x=4.$$

10. $\frac{4x}{x+1}-\frac{x}{x-2}=3.$

$$\frac{4x}{x+1}-\frac{x}{x-2}=3.$$

Multiply by the L. C. D., $(x+1)(x-2)$.

$$4x(x-2)-x(x+1)=3(x+1)(x-2)$$
$$4x^2-8x-x^2-x=3x^2-3x-6$$
$$4x^2-x^2-3x^2-8x-x+3x=-6$$
$$-6x=-6.$$
$$\therefore x=1.$$

11. $\frac{2x+1}{4}-\frac{4x-1}{10}+1\frac{1}{4}=0.$

$$\frac{2x+1}{4}-\frac{4x-1}{10}+1\frac{1}{4}=0.$$

Multiply by the L. C. D., 20.

$$10x+5-8x+2+25=0$$
$$10x-8x=-25-5-2$$
$$2x=-32.$$
$$\therefore x=-16.$$

12. $\dfrac{x-1}{5}-\dfrac{43-5x}{6}-\dfrac{3x-1}{8}=0.$

$$\frac{x-1}{5}-\frac{43-5x}{6}-\frac{3x-1}{8}=0.$$

Multiply by the L. C. D., 120.

$$24x-24-860+100x-45x+15=0$$
$$24x+100x-45x=24+860-15$$
$$79x=869.$$
$$\therefore x=11.$$

13. $\dfrac{1}{x+7}=\dfrac{2}{x+1}-\dfrac{1}{x+3}.$

$$\frac{1}{x+7}=\frac{2}{x+1}-\frac{1}{x+3}.$$

Multiply by the L. C. D., $(x+1)(x+3)(x+7)$.

$$x^2+4x+3=2x^2+20x+42-x^2-8x-7$$
$$x^2-2x^2+x^2+4x-20x+8x=42-7-3$$
$$-8x=32.$$
$$\therefore x=-4.$$

14. $\dfrac{1}{x+4}+\dfrac{2}{x+6}-\dfrac{3}{x+5}=0.$

$$\frac{1}{x+4}+\frac{2}{x+6}-\frac{3}{x+5}=0.$$

Multiply by the L. C. D., $(x+4)(x+5)(x+6)$.

$$x^2+11x+30+2x^2+18x+40-3x^2-30x-72=0$$
$$x^2+2x^2-3x^2+11x+18x-30x=72-30-40$$
$$-x=2.$$
$$\therefore x=-2.$$

15. $\dfrac{4}{x^2-1}+\dfrac{1}{x-1}+\dfrac{1}{x+1}=0.$

$$\frac{4}{x^2-1}+\frac{1}{x-1}+\frac{1}{x+1}=0.$$

Multiply by the L. C. D., x^2-1.

$$4+x+1+x-1=0$$
$$x+x=-4-1+1$$
$$2x=-4.$$
$$\therefore x=-2.$$

16. $\frac{3x+1}{4} - \frac{5x-4}{7} = 12 - 2x - \frac{x-2}{3}$.

$$\frac{3x+1}{4} - \frac{5x-4}{7} = 12 - 2x - \frac{x-2}{3}.$$

Multiply by the L. C. D., 84.

$$63x + 21 - 60x + 48 = 1008 - 168x - 28x + 56$$
$$63x - 60x + 168x + 28x = 1008 + 56 - 21 - 48$$
$$199x = 995.$$
$$\therefore x = 5.$$

17. $\frac{5}{8}(5x+3) - \frac{1}{3}(3-4x) + \frac{1}{6}(9-5x) = \frac{1}{2}(31-x)$.

$$\frac{5}{8}(5x+3) - \frac{1}{3}(3-4x) + \frac{1}{6}(9-5x) = \frac{1}{2}(31-x).$$

Multiply by the L. C. D., 24.

$$15x + 9 - 24 + 32x + 36 - 20x = 372 - 12x$$
$$15x + 32x - 20x + 12x = 372 - 9 + 24 - 36$$
$$39x = 351.$$
$$\therefore x = 9.$$

18. $\frac{1}{15}(34x - 56) - \frac{1}{5}(7x-3) - \frac{1}{3}(7x-5) = 0$.

$$\frac{1}{15}(34x - 56) - \frac{1}{5}(7x-3) - \frac{1}{3}(7x-5) = 0.$$

Multiply by the L. C. D., 15.

$$34x - 56 - 21x + 9 - 35x + 25 = 0$$
$$34x - 21x - 35x = 56 - 9 - 25$$
$$-22x = 22.$$
$$\therefore x = -1.$$

Exercise 52. Page 106.

Solve:

1. $\frac{2}{3}(x+1) - \frac{1}{7}(x+5) = 1$.

$$\frac{2}{3}(x+1) - \frac{1}{7}(x+5) = 1.$$

Multiply by the L. C. D., 21.

$$14x + 14 - 3x - 15 = 21$$
$$14x - 3x = 21 - 14 + 15$$
$$11x = 22.$$
$$\therefore x = 2.$$

2. $\frac{6}{7}(x-9)-\frac{1}{3}(5-x)+3x+1=0.$

$$\frac{6}{7}(x-9)-\frac{1}{3}(5-x)+3x+1=0.$$

Multiply by the L. C. D., 21.

$$18x-162-35+7x+63x+21=0$$
$$18x+7x+63x=162+35-21$$
$$88x=176.$$
$$\therefore x=2.$$

3. $\frac{1}{3}(5x-24)+\frac{1}{7}(x-2)-2(x-1)=0.$

$$\frac{1}{3}(5x-24)+\frac{1}{7}(x-2)-2(x-1)=0.$$

Multiply by the L. C. D., 21.

$$35x-168+3x-6-42x+42=0$$
$$35x+3x-42x=168+6-42$$
$$-4x=132.$$
$$\therefore x=-33.$$

4. $\frac{x+3}{4}+\frac{7x-2}{5}=\frac{5x-1}{4}+\frac{5x+4}{9}.$

$$\frac{x+3}{4}+\frac{7x-2}{5}=\frac{5x-1}{4}+\frac{5x+4}{9}.$$

Multiply by the L. C. D., 180.

$$45x+135+252x-72=225x-45+100x+80$$
$$45x+252x-225x-100x=80-45+72-135$$
$$-28x=-28.$$
$$\therefore x=1.$$

5. $\frac{x+1}{3}-\frac{x-1}{4}=\frac{x-2}{5}-\frac{x-3}{6}+\frac{31}{60}.$

$$\frac{x+1}{3}-\frac{x-1}{4}=\frac{x-2}{5}-\frac{x-3}{6}+\frac{31}{60}.$$

Multiply by the L. C. D., 60.

$$20x+20-15x+15=12x-24-10x+30+31$$
$$20x-15x-12x+10x=30+31-20-15-24$$
$$3x=2.$$
$$\therefore x=\tfrac{2}{3}.$$

6. $\dfrac{(2x-1)(2-x)}{2} + x^2 - \dfrac{1+3x}{2} = 0.$

$$\frac{(2x-1)(2-x)}{2} + x^2 - \frac{1+3x}{2} = 0.$$

Multiply by the L. C. D., 2.

$$-2x^2 + 5x - 2 + 2x^2 - 1 - 3x = 0.$$
$$-2x^2 + 2x^2 + 5x - 3x = 2 + 1$$
$$2x = 3.$$
$$\therefore x = 1\tfrac{1}{2}.$$

7. $\dfrac{6x-11}{4} - \dfrac{3-4x}{6} = \dfrac{4}{3} - \dfrac{x}{8}.$

$$\frac{6x-11}{4} - \frac{3-4x}{6} = \frac{4}{3} - \frac{x}{8}.$$

Multiply by the L. C. D., 24.

$$36x - 66 - 12 + 16x = 32 - 3x$$
$$36x + 16x + 3x = 32 + 66 + 12$$
$$55x = 110.$$
$$\therefore x = 2.$$

8. $\dfrac{x+6}{4} - \dfrac{16-3x}{12} = 4\tfrac{1}{6}.$

$$\frac{x+6}{4} - \frac{16-3x}{12} = 4\tfrac{1}{6}.$$

Multiply by the L. C. D., 12.

$$3x + 18 - 16 + 3x = 50$$
$$3x + 3x = 50 - 18 + 16$$
$$6x = 48.$$
$$\therefore x = 8.$$

9. $x - \dfrac{x-2}{3} = \dfrac{x+23}{4} - \dfrac{10+x}{5}.$

$$x - \frac{x-2}{3} = \frac{x+23}{4} - \frac{10+x}{5}.$$

Multiply by the L. C. D., 60.

$$60x - 20x + 40 = 15x + 345 - 120 - 12x.$$
$$60x - 20x - 15x + 12x = 345 - 120 - 40$$
$$37x = 185.$$
$$\therefore x = 5.$$

10. $\dfrac{5x+3}{x-1}+\dfrac{2x-3}{2x-1}=6.$

$$\frac{5x+3}{x-1}+\frac{2x-3}{2x-1}=6.$$

Multiply by the L. C. D., $(x-1)(2x-1)$.

$$10x^2+x-3+2x^2-5x+3=12x^2-18x+6$$
$$10x^2+2x^2-12x^2+x-5x+18x=6-3+3$$
$$14x=6.$$
$$\therefore x=\tfrac{3}{7}.$$

11. $\dfrac{3x}{4x+1}+1=2-\dfrac{x}{2(2x-1)}.$

$$\frac{3x}{4x+1}+1=2-\frac{x}{2(2x-1)}.$$

Multiply by the L. C. D., $2(2x-1)(4x+1)$.

$$12x^2-6x+16x^2-4x-2=32x^2-8x-4-4x^2-x$$
$$12x^2+16x^2-32x^2+4x^2-6x-4x+8x+x=-4+2$$
$$-x=-2.$$
$$\therefore x=2.$$

12. $\dfrac{8x+7}{5x+4}-1=1-\dfrac{2x}{5x+1}.$

$$\frac{8x+7}{5x+4}-1=1-\frac{2x}{5x+1}.$$

Multiply by the L. C. D., $(5x+4)(5x+1)$.

$$40x^2+43x+7-25x^2-25x-4=25x^2+25x+4-10x^2-8x$$
$$40x^2-25x^2-25x^2+10x^2+43x-25x-25x+8x=4-7+4.$$
$$\therefore x=1.$$

13. $\dfrac{x+1}{2(x-1)}-\dfrac{x-1}{x+1}=\dfrac{17-x^2}{2(x^2-1)}.$

$$\frac{x+1}{2(x-1)}-\frac{x-1}{x+1}=\frac{17-x^2}{2(x^2-1)}.$$

Multiply by the L. C. D., $2(x^2-1)$.

$$x^2+2x+1-2x^2+4x-2=17-x^2$$
$$x^2-2x^2+x^2+2x+4x=17-1+2$$
$$6x=18.$$
$$\therefore x=3.$$

Exercise 53. Page 107.

1. $\dfrac{10x+13}{18}-\dfrac{x+2}{x-3}=\dfrac{5x-4}{9}.$

$$\frac{10x+13}{18}-\frac{x+2}{x-3}=\frac{5x-4}{9}.$$

Multiply by 18.

$$10x+13-\frac{18(x+2)}{x-3}=10x-8$$

$$10x-10x+13+8=\frac{18(x+2)}{x-3}$$

$$21=\frac{18(x+2)}{x-3}.$$

Divide by 3, $$7=\frac{6(x+2)}{x-3}$$

$$\therefore 7x-21=6x+12$$

$$7x-6x=21+12.$$

$$\therefore x=33.$$

2. $\dfrac{6x+7}{10}-\dfrac{3x+1}{5}=\dfrac{x-1}{3x-4}.$

$$\frac{6x+7}{10}-\frac{3x+1}{5}=\frac{x-1}{3x-4}.$$

Multiply by 10.

$$6x+7-6x-2=\frac{10(x-1)}{3x-4}$$

$$5=\frac{10(x-1)}{3x-4}.$$

Divide by 5, $$1=\frac{2(x-1)}{3x-4}$$

$$\therefore 3x-4=2x-2$$

$$3x-2x=4-2.$$

$$\therefore x=2.$$

3. $\frac{11x-12}{14}-\frac{11x-7}{19x+7}=\frac{22x-36}{28}.$

$$\frac{11x-12}{14}-\frac{11x-7}{19x+7}=\frac{22x-36}{28}.$$

Multiply by 28.

$$22x-24-\frac{28(11x-7)}{19x+7}=22x-36$$

$$22x-22x-24+36=\frac{28(11x-7)}{19x+7}$$

$$12=\frac{28(11x-7)}{19x+7}.$$

Divide by 4, $\qquad 3=\frac{7(11x-7)}{19x+7}$

$$\therefore 57x+21=77x-49$$

$$57x-77x=-49-21$$

$$-20x=-70.$$

$$\therefore x=3\tfrac{1}{2}.$$

4. $\frac{2x-1}{5}+\frac{2x-3}{17x-12}=\frac{4x-3}{10}.$

$$\frac{2x-1}{5}+\frac{2x-3}{17x-12}=\frac{4x-3}{10}.$$

Multiply by 10.

$$4x-2+\frac{10(2x-3)}{17x+12}=4x-3.$$

$$4x-4x-2+3=-\frac{10(2x-3)}{17x-12}$$

$$1=-\frac{10(2x-3)}{17x-12}$$

$$17x-12=-20x+30$$

$$17x+20x=30+12$$

$$37x=42.$$

$$\therefore x=1\tfrac{5}{37}.$$

5. $\frac{11x-13}{7}-\frac{13x+7}{3x+7}=\frac{22x-75}{14}.$

$$\frac{11x-13}{7}-\frac{13x+7}{3x+7}=\frac{22x-75}{14}.$$

Multiply by 14.

$$22x-26-\frac{14(13x+7)}{3x+7}=22x-75.$$

$$22x-22x-26+75=\frac{14(13x+7)}{3x+7}$$

$$49=\frac{14(13x+7)}{3x+7}$$

Divide by 7, $\quad 7=\dfrac{2(13x+7)}{3x+7}$

$$\therefore 21x+49=26x+14$$

$$21x-26x=14-49$$

$$-5x=-35.$$

$$\therefore x=7.$$

6. $\dfrac{6x-13}{2x+3}+\dfrac{6x+7}{9}-\dfrac{2x+4}{3}=0.$

$$\frac{6x-13}{2x+3}+\frac{6x+7}{9}-\frac{2x+4}{3}=0.$$

Multiply by 9.

$$\frac{9(6x-13)}{2x+3}+6x+7-6x-12=0.$$

$$\frac{9(6x-13)}{2x+3}=5.$$

$$54x-117=10x+15$$

$$54x-10x=117+15$$

$$44x=132.$$

$$\therefore x=3.$$

Exercise 54. Page 108.

Solve:

1. $a(x-a)=b(x-b).$

$$a(x-a)=b(x-b).$$

$$ax-a^2=bx-b^2$$

$$ax-bx=a^2-b^2$$

$$x(a-b)=(a+b)(a-b).$$

$$\therefore x=a+b.$$

2. $(a+b)x+(a-b)x=a^2.$

$$(a+b)x+(a-b)x=a^2$$

$$ax+bx+ax-bx=a^2$$

$$2ax=a^2.$$

$$\therefore x=\frac{a^2}{2a}=\frac{a}{2}.$$

3. $(a+b)x-(a-b)x=b^2$.

$$(a+b)x-(a-b)x=b^2.$$
$$ax+bx-ax+bx=b^2$$
$$2bx=b^2.$$
$$\therefore x=\frac{b^2}{2b}=\frac{b}{2}.$$

4. $(2x-a)+(x-2a)=3a$.

$$(2x-a)+(x-2a)=3a.$$
$$2x-a+x-2a=3a$$
$$2x+x=3a+a+2a$$
$$3x=6a.$$
$$\therefore x=2a.$$

5. $(x+a+b)+(x+a-b)=2b$.

$$(x+a+b)+(x+a-b)=2b.$$
$$x+a+b+x+a-b=2b$$
$$x+x=2b-a-a-b+b$$
$$2x=2b-2a.$$
$$\therefore x=b-a.$$

6. $(x-a)(x-b)=x(x+c)$.

$$(x-a)(x-b)=x(x+c).$$
$$x^2-ax-bx+ab=x^2+cx$$
$$x^2-x^2-ax-bx-cx=-ab$$
$$ax+bx+cx=ab$$
$$(a+b+c)x=ab.$$
$$\therefore x=\frac{ab}{a+b+c}.$$

7. $x^2+b^2=(a-x)(a-x)$.

$$x^2+b^2=(a-x)(a-x).$$
$$x^2+b^2=a^2-2ax+x^2$$
$$x^2-x^2+2ax=a^2-b^2$$
$$2ax=a^2-b^2.$$
$$\therefore x=\frac{a^2-b^2}{2a}.$$

8. $(a+b)(2-x)=(a-b)(2+x)$.

$$(a+b)(2-x)=(a-b)(2+x).$$
$$2a+2b-ax-bx=2a-2b+ax-bx$$
$$-ax-bx-ax+bx=2a-2b-2a-2b$$
$$-2ax=-4b.$$
$$\therefore x=\frac{4b}{2a}=\frac{2b}{a}.$$

9. $(x-a)(2x-a)=2(x-b)^2.$

$(x-a)(2x-a)=2(x-b)^2.$

$$2x^2-3ax+a^2=2x^2-4bx+2b^2$$
$$2x^2-2x^2-3ax+4bx=2b^2-a^2$$
$$x(4b-3a)=2b^2-a^2$$
$$x=\frac{2b^2-a^2}{4b-3a}.$$

10. $(a+bx)(c+d)=(a+b)(c+dx).$

$(a+bx)(c+d)=(a+b)(c+dx).$

$$ac+bcx+ad+bdx=ac+adx+bc+bdx.$$
$$bcx+bdx-adx-bdx=ac-ac+bc-ad$$
$$x(bc-ad)=bc-ad.$$
$$\therefore x=1.$$

11. $\dfrac{x}{a-b}-\dfrac{3a}{a+b}=\dfrac{bx}{a^2-b^2}.$

$\dfrac{x}{a-b}-\dfrac{3a}{a+b}=\dfrac{bx}{a^2-b^2}.$

Multiply by the L. C. D., a^2-b^2.

$$ax+bx-3a^2+3ab=bx$$
$$ax+bx-bx=3a^2-3ab$$
$$ax=3a(a-b).$$
$$\therefore x=3(a-b).$$

Exercise 55. Page 109.

1. The difference between the fifth and seventh parts of a certain number is 2. Find the number.

Let $x=$ the number.

Then $\frac{x}{5}-\frac{x}{7}=$ the difference of its fifth and seventh parts.

But $2=$ the difference of its fifth and seventh parts.

$$\therefore \frac{x}{5}-\frac{x}{7}=2$$
$$7x-5x=70$$
$$2x=70.$$
$$\therefore x=35.$$

2. One-half of a certain number exceeds the sum of its fifth and seventh parts by 11. Find the number.

Let $x =$ the number.

Then $\frac{x}{2} =$ one-half the number,

$\frac{x}{5} + \frac{x}{7} =$ the sum of its fifth and seventh parts,

$\frac{x}{2} - \left(\frac{x}{5} + \frac{x}{7}\right) =$ the given excess.

But $11 =$ the given excess.

$$\therefore \frac{x}{2} - \left(\frac{x}{5} + \frac{x}{7}\right) = 11$$

$$35x - 14x - 10x = 770$$

$$11x = 770.$$

$$\therefore x = 70.$$

3. The sum of the third and sixth parts of a certain number exceeds the difference of its sixth and ninth parts by 16. Find the number.

Let $x =$ the number.

Then $\frac{x}{3} + \frac{x}{6} =$ the sum of its third and sixth parts,

$\frac{x}{6} - \frac{x}{9} =$ the difference of its sixth and ninth parts.

$\left(\frac{x}{3} + \frac{x}{6}\right) - \left(\frac{x}{6} - \frac{x}{9}\right) =$ the given excess.

But $16 =$ the given excess.

$$\therefore \frac{x}{3} + \frac{x}{6} - \left(\frac{x}{6} - \frac{x}{9}\right) = 16$$

$$6x + 3x - 3x + 2x = 288$$

$$8x = 288.$$

$$\therefore x = 36.$$

4. There are two consecutive numbers, x and $x + 1$, such that one-half of the larger exceeds one-third of the smaller number by 10. Find the numbers.

Let x and $x + 1 =$ the numbers.

Then $\frac{x+1}{2} =$ one-half the larger,

$\frac{x}{3} =$ one-third the smaller,

$$\frac{x+1}{2} - \frac{x}{3} = \text{the given excess,}$$

But $\qquad 10 = \text{the given excess.}$

$$\therefore \frac{x+1}{2} - \frac{x}{3} = 10$$

$$3x + 3 - 2x = 60$$

$$3x - 2x = 60 - 3.$$

$$\therefore x = 57$$

and $\qquad x + 1 = 58.$

$\therefore$ the numbers are 57 and 58.

Exercise 56. Page 110.

1. The sum of two numbers is 100, and if the greater is divided by the smaller number, the quotient is 4 and the remainder 5. Find the numbers.

Let $\qquad x = \text{the greater number.}$

Then $\qquad 100 - x = \text{the smaller number,}$

$$\frac{x-5}{100-x} = 4$$

$$x - 5 = 400 - 4x$$

$$x + 4x = 400 + 5$$

$$5x = 405.$$

$$\therefore x = 81,$$

and $\qquad 100 - x = 19.$

$\therefore$ the numbers are 81 and 19.

2. The sum of two numbers is 124, and if the greater is divided by the smaller number, the quotient is 4 and the remainder 4. Find the numbers.

Let $\qquad x = \text{the greater number.}$

Then $\qquad 124 - x = \text{the smaller number.}$

$$\frac{x-4}{124-x} = 4$$

$$x - 4 = 496 - 4x$$

$$x + 4x = 496 + 4$$

$$5x = 500.$$

$$\therefore x = 100,$$

and $\qquad 124 - x = 24.$

$\therefore$ the numbers are 100 and 24.

3. The difference of two numbers is 49, and if the greater is divided by the smaller, the quotient is 4 and the remainder 4. Find the numbers.

Let $x =$ the greater number.

Then $x - 49 =$ the smaller number.

$$\frac{x-4}{x-49} = 4$$

$$x - 4 = 4x - 196$$

$$x - 4x = -196 + 4$$

$$-3x = -192.$$

$$\therefore x = 64,$$

and $x - 49 = 15.$

$\therefore$ the numbers are 64 and 15.

4. The difference of two numbers is 91, and if the greater is divided by the smaller, the quotient is 8 and the remainder 7. Find the numbers.

Let $x =$ the greater number.

Then $x - 91 =$ the smaller number.

$$\frac{x-7}{x-91} = 8$$

$$x - 7 = 8x - 728$$

$$x - 8x = 7 - 728$$

$$-7x = -721.$$

$$\therefore x = 103,$$

and $x - 91 = 12.$

$\therefore$ the numbers are 103 and 12.

5. Divide 320 into two parts such that the smaller part is contained in the larger part 11 times, with a remainder of 20.

Let $x =$ the larger part.

Then $320 - x =$ the smaller part.

$$\frac{x-20}{320-x} = 11$$

$$x - 20 = 3520 - 11x$$

$$x + 11x = 3520 + 20$$

$$12x = 3540.$$

$$\therefore x = 295,$$

and $320 - x = 25.$

$\therefore$ the numbers are 295 and 25.

Exercise 57. Page 111.

1. A son is one-fourth as old as his father. In 24 years he will be one-half as old. Find the age of the son.

Let $x =$ the number of years in the son's age.
Then $4x =$ the number of years in the father's age.
$x + 24 =$ the number of years in the son's age in 24 years,
$4x + 24 =$ the number of years in the father's age in 24 years.

$$\therefore x + 24 = \tfrac{1}{2}(4x + 24)$$
$$x + 24 = 2x + 12$$
$$x - 2x = 12 - 24.$$
$$\therefore x = 12, \text{ son's age.}$$

2. B's age is one-sixth of A's age. In 15 years B's age will be one-third of A's age. Find their ages.

Let $x =$ the number of years in B's age.
Then $6x =$ the number of years in A's age.
$x + 15 =$ the number of years in B's age 15 years hence,
$6x + 15 =$ the number of years in A's age 15 years hence.

$$\therefore x + 15 = \tfrac{1}{3}(6x + 15)$$
$$x + 15 = 2x + 5.$$
$$\therefore x = 10, \text{ B's age,}$$
and
$$6x = 60, \text{ A's age.}$$

3. The sum of the ages of A and B is 30 years, and 5 years hence B's age will be one-third of A's. Find their ages.

Let $x =$ the number of years in A's age.
Then $30 - x =$ the number of years in B's age.
$x + 5 =$ the number of years in A's age 5 years hence,
and $30 - x + 5 =$ the number of years in B's age 5 years hence.

$$\therefore 30 - x + 5 = \tfrac{1}{3}(x + 5)$$
$$90 - 3x + 15 = x + 5$$
$$-3x - x = 5 - 90 - 15$$
$$-4x = -100.$$
$$\therefore x = 25, \text{ A's age,}$$
and
$$30 - x = 5, \text{ B's age.}$$

4. A father is 35 years old, and his son is one-fourth of that age. In how many years will the son be half as old as his father?

Let $x =$ the number of years.

Then $35 + x =$ the number of years in father's age 5 years hence,

and $\frac{35}{4} + x =$ the number of years in son's age 5 years hence.

$$\therefore \tfrac{35}{4} + x = \tfrac{1}{2}(35 + x)$$
$$35 + 4x = 70 + 2x$$
$$4x - 2x = 70 - 35$$
$$2x = 35.$$
$$\therefore x = 17\tfrac{1}{2} \text{ years.}$$

5. A is 60 years old, and B's age is two-thirds of A's. How many years ago was B's age one-fifth of A's?

Let $x =$ the number of years.

$\frac{2}{3}$ of $60 = 40$, B's age at present.

$60 - x =$ the number of years old A was x years ago,

and $40 - x =$ the number of years old B was x years ago.

$$\therefore 40 - x = \tfrac{1}{5}(60 - x)$$
$$200 - 5x = 60 - x$$
$$-5x + x = 60 - 200$$
$$-4x = -140.$$
$$\therefore x = 35.$$

6. A son is one-third as old as his father. Four years ago he was only one-fourth as old as his father. What is the age of each?

Let $x =$ the number of years in father's age.

Then $\dfrac{x}{3} =$ the number of years in son's age.

$x - 4 =$ the number of years in father's age 4 years ago,

and $\dfrac{x}{3} - 4 =$ the number of years in son's age 4 years ago.

$$\therefore \frac{x}{3} - 4 = \frac{1}{4}(x - 4)$$
$$4x - 48 = 3x - 12$$
$$4x - 3x = 48 - 12.$$
$$\therefore x = 36, \text{ father's age,}$$

and $\frac{36}{3} = 12$, son's age.

7. A is 50 years old, and B is half as old as A. In how many years will B be two-thirds as old as A?

$\frac{1}{2}$ of $50 = 25$, B's age.

Let $x =$ the number of years,

$50 + x =$ the number of years in A's age x years hence,

and $25 + x =$ the number of years in B's age x years hence.

$$\therefore 25 + x = \tfrac{2}{3}(50 + x)$$
$$75 + 3x = 100 + 2x$$
$$3x - 2x = 100 - 75.$$
$$\therefore x = 25.$$

8. B is one-half as old as A. Ten years ago he was one-fourth as old as A. What are their present ages?

Let $x =$ the number of years in A's age.

Then $\frac{x}{2} =$ the number of years in B's age,

$x - 10 =$ the number of years in A's age 10 years ago,

and $\frac{x}{2} - 10 =$ the number of years in B's age 10 years ago.

$$\therefore \frac{x}{2} - 10 = \tfrac{1}{4}(x - 10)$$
$$2x - 40 = x - 10$$
$$2x - x = 40 - 10.$$
$$\therefore x = 30, \text{ A's age,}$$

and $\frac{x}{2} = 15$, B's age.

9. The sum of the ages of a father and his son is 80 years. The son's age increased by 5 years is one-fourth of the father's age. Find their ages.

Let $x =$ the number of years in son's age.

Then $80 - x =$ the number of years in father's age,

$x + 5 =$ the number of years in son's age increased by 5.

$$\therefore x + 5 = \tfrac{1}{4}(80 - x)$$
$$4x + 20 = 80 - x$$
$$4x + x = 80 - 20$$
$$5x = 60.$$
$$\therefore x = 12, \text{ son's age,}$$

and $80 - x = 68$, father's age.

Exercise 58. Page 112.

1. A can do a piece of work in 3 days, B in 5 days, and C in 6 days. How long will it take them to do it working together?

Let $x =$ the number of days it will take all together.

Then $\frac{1}{x} =$ the part all together can do in one day.

$\frac{1}{3} =$ the part A can do in one day,

$\frac{1}{5} =$ the part B can do in one day,

$\frac{1}{6} =$ the part C can do in one day,

and $\frac{1}{3} + \frac{1}{5} + \frac{1}{6} =$ the part all together can do in one day,

$$\therefore \frac{1}{3} + \frac{1}{5} + \frac{1}{6} = \frac{1}{x},$$

$$10x + 6x + 5x = 30$$

$$21x = 30.$$

$$\therefore x = 1\tfrac{9}{30} = 1\tfrac{3}{10} \text{ days.}$$

2. A can do a piece of work in 5 days, B in 4 days, and C in 3 days. How long will it take them together to do the work?

Let $x =$ the number of days it will take all together.

Then $\frac{1}{x} =$ the part all together can do in one day.

$\frac{1}{5} =$ the part A can do in one day.

$\frac{1}{4} =$ the part B can do in one day,

$\frac{1}{3} =$ the part C can do in one day.

and $\frac{1}{5} + \frac{1}{4} + \frac{1}{3} =$ the part all together can do in one day.

$$\therefore \frac{1}{5} + \frac{1}{4} + \frac{1}{3} = \frac{1}{x}.$$

$$12x + 15x + 20x = 60,$$

$$47x = 60.$$

$$\therefore x = 1\tfrac{13}{47} \text{ days.}$$

3. A can do a piece of work in $2\frac{1}{2}$ days, B in $3\frac{1}{3}$ days, and C in $3\frac{3}{4}$ days. How long will it take them together to do the work?

Let $x =$ the number of days it will take all together.

Then $\frac{1}{x} =$ the part all together can do in one day,

$$2\tfrac{1}{2} = \frac{5}{2} \text{ and } \frac{1}{\frac{5}{2}} = \frac{2}{5}$$

$$3\tfrac{1}{2} = \frac{7}{2} \text{ and } \frac{1}{\frac{7}{2}} = \frac{2}{7}$$

$$3\tfrac{3}{4} = \frac{15}{4} \text{ and } \frac{1}{\frac{15}{4}} = \frac{4}{15}$$

$$\tfrac{2}{5} = \text{the part A can do in one day,}$$

$$\tfrac{2}{7} = \text{the part B can do in one day,}$$

$$\tfrac{4}{15} = \text{the part C can do in one day,}$$

$$\tfrac{2}{5} + \tfrac{2}{7} + \tfrac{4}{15} = \text{the part all together can do in one day.}$$

$$\therefore \frac{2}{5} + \frac{2}{7} + \frac{4}{15} = \frac{1}{x}$$

$$42x + 30x + 28x = 105$$

$$100x = 105.$$

$$\therefore x = 1\tfrac{5}{100} = 1\tfrac{1}{20} \text{ days.}$$

4. A can do a piece of work in 10 days, B in 12 days; A and B together, with the help of C, can do the work in 4 days. How long will it take C alone to do the work?

Let $x =$ the number of days it will take C alone.

Then $\frac{1}{x} =$ the part C can do in one day,

$$\tfrac{1}{10} = \text{the part A can do in one day,}$$

$$\tfrac{1}{12} = \text{the part B can do in one day,}$$

$$\frac{1}{x} + \frac{1}{10} + \frac{1}{12} = \text{the part they can all do in one day.}$$

But $\tfrac{1}{4} =$ the part they can all do in one day.

$$\therefore \frac{1}{x} + \frac{1}{10} + \frac{1}{12} = \frac{1}{4}$$

$$60 + 6x + 5x = 15x$$

$$6x + 5x - 15x = -60$$

$$-4x = -60.$$

$$\therefore x = 15 \text{ days.}$$

5. A and B together can mow a field in 10 hours, A and C in 12 hours, and A alone in 20 hours. In what time can B and C together mow the field?

Let $x =$ the number of hours it takes B and C together.

Then $\frac{1}{x} =$ the part B and C can do in one hour,

$$\tfrac{1}{10} = \text{the part A and B can do in one hour,}$$

$$\tfrac{1}{12} = \text{the part A and C can do in one hour,}$$

$\frac{1}{20}$ = the part A can do alone in one hour,

$\frac{1}{10} - \frac{1}{20}$ = the part B can do alone in one hour,

$\frac{1}{12} - \frac{1}{20}$ = the part C can do alone in one hour,

$\therefore (\frac{1}{10} - \frac{1}{20}) + (\frac{1}{12} - \frac{1}{20})$ = the part B and C can do in one hour.

But $\frac{1}{x}$ = the part B and C can do in 1 hour.

$$\therefore \left(\frac{1}{10} - \frac{1}{20}\right) + \left(\frac{1}{12} - \frac{1}{20}\right) = \frac{1}{x}$$

$$6x - 3x + 5x - 3x = 60$$

$$5x = 60.$$

$$\therefore x = 12.$$

6. A and B together can build a wall in 12 days, A and C in 15 days, B and C in 20 days. In what time can they build the wall if they all work together?

Let x = the number of days it will take all together.

Then $\frac{1}{x}$ = the part all together can do in one day,

$\frac{2}{x}$ = the part all together can do in two days,

$\frac{1}{12} + \frac{1}{15} + \frac{1}{20}$ = the part all together can do in two days.

$$\therefore \frac{1}{12} + \frac{1}{15} + \frac{1}{20} = \frac{2}{x}$$

$$5x + 4x + 3x = 120$$

$$12x = 120.$$

$$\therefore x = 10.$$

Exercise 59. Page 113.

1. A cistern can be filled by three pipes in 16, 24, and 32 hours, respectively. In what time will it be filled by all the pipes together?

Let x = the number of hours it will take all together.

Then $\frac{1}{x}$ = the part all together can fill in one hour,

$\frac{1}{16} + \frac{1}{24} + \frac{1}{32}$ = the part all together can fill in one hour.

$$\therefore \frac{1}{16} + \frac{1}{24} + \frac{1}{32} = \frac{1}{x}$$

$$6x + 4x + 3x = 96$$

$$13x = 96.$$

$$\therefore x = 7\tfrac{5}{13} \text{ hours.}$$

2. A tank can be filled by two pipes in 3 hours and 4 hours, respectively, and can be emptied by a third pipe in 6 hours. In what time will the cistern be filled if the pipes are all running together?

Let $x =$ the number of hours it will take all together.

Then $\frac{1}{x} =$ the part all together can fill in one hour,

$\frac{1}{3} + \frac{1}{4} - \frac{1}{6} =$ the part all together can fill in one hour.

$$\therefore \frac{1}{3} + \frac{1}{4} - \frac{1}{6} = \frac{1}{x}$$

$$4x + 3x - 2x = 12$$

$$5x = 12.$$

$$\therefore x = 2\tfrac{2}{5} \text{ hours.}$$

3. A tank can be filled by three pipes in 1 hour and 40 minutes, 3 hours and 20 minutes, and 5 hours, respectively. In what time will the tank be filled if all three pipes are running together?

1 hour and 40 minutes $= 1\frac{2}{3}$ hours.

3 hours and 20 minutes $= 3\frac{1}{3}$ hours.

Let $x =$ the number of hours it will take all together.

Then $\frac{1}{x} =$ the part all together can do in one hour,

$\frac{3}{5} + \frac{3}{10} + \frac{1}{5} =$ the part all together can do in one hour.

$$\therefore \frac{3}{5} + \frac{3}{10} + \frac{1}{5} = \frac{1}{x}$$

$$6x + 3x + 2x = 10$$

$$11x = 10.$$

$$\therefore x = \tfrac{10}{11} \text{ hours.}$$

4. A cistern can be filled by three pipes in $2\frac{1}{3}$ hours, $3\frac{1}{2}$ hours, and $4\frac{2}{3}$ hours, respectively. In what time will the cistern be filled if all the pipes are running together?

Let $x =$ the number of hours it will take all together.

Then $\frac{1}{x} =$ the part all together can do in one hour,

$\frac{3}{7} + \frac{2}{7} + \frac{3}{14} =$ the part all together can do in one hour.

$$\therefore \frac{3}{7} + \frac{2}{7} + \frac{3}{14} = \frac{1}{x}$$

$$6x + 4x + 3x = 14$$

$$13x = 14.$$

$$\therefore x = 1\tfrac{1}{13} \text{ hours.}$$

5. A cistern has three pipes. The first pipe will fill the cistern in 12 hours, the second in 20 hours, and all three pipes together will fill it in 6 hours. How long will it take the third pipe alone to fill it?

Let $x =$ the number of hours it takes the third pipe.

Then $\frac{1}{x} =$ the part the third pipe fills in one hour.

and $\frac{1}{12} + \frac{1}{20} + \frac{1}{x} =$ the part all together will fill in one hour.

But $\frac{1}{6} =$ the part all together will fill in one hour.

$$\therefore \frac{1}{12} + \frac{1}{20} + \frac{1}{x} = \frac{1}{6}$$

$$5x + 3x + 60 = 10x$$

$$5x + 3x - 10x = -60$$

$$-2x = -60.$$

$$\therefore x = 30 \text{ hours.}$$

Exercise 60. Page 114.

1. A sets out from Boston and walks towards Portland at the rate of 3 miles an hour. Three hours afterward B sets out from the same place and walks in the same direction at the rate of 4 miles an hour. How far from Boston will B overtake A?

Let $x =$ the number of hours the first walks.

Then $x - 3 =$ the number of hours the second walks,

$3x =$ the number of miles the first walks,

$(x - 3)\,4 =$ the number of miles the second walks.

They both travel the same distance.

$$\therefore 3x = (x - 3)\,4$$

$$3x = 4x - 12$$

$$3x - 4x = -12.$$

$$\therefore x = 12$$

and $3x = 36$, number of miles from Boston.

2. A courier who goes at the rate of $6\frac{1}{2}$ miles an hour is followed, after 4 hours, by another who goes at the rate of $7\frac{1}{2}$ miles an hour. In how many hours will the second overtake the first?

Let $x =$ the number of hours the first travels.

Then $x - 4 =$ the number of hours the second travels,

$\frac{13x}{2} =$ the number of miles the first travels,

$7\frac{1}{2}(x-4) =$ the number of miles the second travels.

They both travel the same distance.

$$\therefore \frac{13x}{2} = 7\tfrac{1}{2}(x-4)$$

$$13x = 15x - 60$$

$$13x - 15x = -60$$

$$-2x = -60.$$

$$\therefore x = 30.$$

Therefore the second courier will overtake the first in

$$30 - 4 = 26 \text{ hours.}$$

3. A person walks to the top of a mountain at the rate of two miles an hour, and down the same way at the rate of 4 miles an hour. If he is out 6 hours, how far is it to the top of the mountain?

Let $x =$ the number of miles to the top of the mountain.

Then $\frac{x}{2} =$ the number of hours it takes to walk up,

and $\frac{x}{4} =$ the number of hours it takes to walk down.

Hence $\frac{x}{2} + \frac{x}{4} =$ the number of hours it takes to walk up and down.

But $6 =$ the number of hours it takes to walk up and down.

$$\therefore \frac{x}{2} + \frac{x}{4} = 6$$

$$2x + x = 24$$

$$3x = 24.$$

$$\therefore x = 8 \text{ miles.}$$

4. In going a certain distance, a train traveling at the rate of 40 miles an hour takes 2 hours less than a train traveling 30 miles an hour. Find the distance.

Let $x =$ the number of miles.

Then $\frac{x}{40} =$ the number of hours required at 40 miles an hour,

and $\frac{x}{30} =$ the number of hours required at 30 miles an hour.

Hence $\frac{x}{30} - \frac{x}{40} =$ the difference in hours.

But $2 =$ the difference in hours.

$$\therefore \frac{x}{30} - \frac{x}{40} = 2$$

$$4x - 3x = 240.$$

$$\therefore x = 240 \text{ miles.}$$

Exercise 61. Page 115.

1. A hound makes 3 leaps while a rabbit makes 5; but 1 of the hound's leaps is equivalent to 2 of the rabbit's. The rabbit has a start of 120 leaps. How many leaps will the rabbit take before she is caught?

Let $3x =$ the number of leaps taken by the hound.
Then $5x =$ the number of leaps taken by the rabbit.
Also, let $a =$ the number of feet in one leap of the rabbit.
Then $2a =$ the number of feet in one leap of the hound.
Therefore $3x \times 2a = 6ax =$ the whole distance.
Also $(120 + 5x)a =$ the whole distance.

$$\therefore 6ax = (120 + 5x)a$$
$$6ax = 120a + 5ax$$

Divide by a,

$$6x = 120 + 5x.$$
$$\therefore x = 120$$

and

$$5x = 600.$$

2. A rabbit takes 6 leaps to a dog's 5, and 7 of the dog's leaps are equivalent to 9 of the rabbit's. The rabbit has a start of 60 of her own leaps. How many leaps must the dog take to catch the rabbit?

Let $6x =$ the number of leaps taken by the rabbit.
Then $5x =$ the number of leaps taken by the dog.
Also, let $a =$ the number of feet in one leap of the rabbit.
Then $\frac{9a}{7} =$ the number of feet in one leap of the dog.
Then $5x \times \frac{9a}{7}$ or $\frac{45ax}{7} =$ the whole distance.
But $(60 + 6x)a =$ the whole distance.

$$\therefore \frac{45ax}{7} = (60 + 6x)a$$
$$45ax = (420 + 42x)a$$

Divide by a,

$$45x - 42x = 420$$
$$3x = 420.$$
$$\therefore x = 140$$
$$5x = 700.$$

3. A dog makes 4 leaps while a rabbit makes 5; but three of the dog's leaps are equivalent to 4 of the rabbit's. The rabbit has a start of 90 of the *dog's leaps*. How many leaps will each take before the rabbit is caught?

Let $4x =$ the number of leaps taken by the dog.

$5x =$ the number of leaps taken by the rabbit.

Also, let $a =$ the number of feet in one leap of the rabbit.

Then $\frac{4a}{3} =$ the number of feet in one leap of the dog.

Then $4x \times \frac{4a}{3}$ or $\frac{16ax}{3} =$ the whole distance.

90 of the dog's leaps = 120 of the rabbit's.

$(120 + 5x)a =$ the whole distance.

$$\therefore \frac{16ax}{3} = (120 + 5x)a$$

$$16ax = (360 + 15x)a$$

Divide by a, $16x - 15x = 360.$

$$\therefore x = 360$$

$4x = 1440$, number of leaps of the dog,

$5x = 1800$, number of leaps of the rabbit.

Exercise 62. Page 116.

1. Find the time between 5 and 6 o'clock when the hands of a clock are together.

At 5 o'clock the hour-hand is 25 minute-spaces ahead of the minute-hand.

Let $x =$ the number of spaces the minute-hand moves over.

Then $x - 25 =$ the number of spaces the hour-hand moves over.

Since the minute-hand moves 12 times as fast as the hour-hand, the minute-hand moves over $12(x - 25)$ spaces.

$$\therefore 12(x - 25) = x$$

$$12x - 300 = x$$

$$12x - x = 300$$

$$11x = 300.$$

$$\therefore x = 27\tfrac{3}{11}.$$

Therefore the time is $27\frac{3}{11}$ minutes past 5 o'clock.

2. Find the time between 2 and 3 o'clock when the hands of a clock are at right angles to each other.

Let $x =$ the number of spaces the minute-hand moves over.

At 2 o'clock the minute-hand is 10 minute-spaces behind the hour-hand, and must be 15 minute-spaces ahead in order to be at right angles.

Then $(x - 10 - 15) =$ the number of spaces the hour-hand moves over.

As the minute-hand moves 12 times as fast as the hour-hand,

$12(x - 10 - 15) =$ the number of spaces the minute-hand moves over.

$$\therefore 12(x - 10 - 15) = x$$
$$12x - 120 - 180 = x$$
$$11x = 300.$$
$$\therefore x = 27\tfrac{3}{11}.$$

Therefore the time is $27\frac{3}{11}$ minutes past 2 o'clock.

3. Find the time between 2 and 3 o'clock when the hands of a clock point in opposite directions.

Let $x =$ the number of spaces the minute-hand moves over.

At 2 o'clock the minute-hand is 10 minute-spaces behind the hour-hand, and must be 30 minute-spaces ahead in order for the hands to point in opposite directions.

Then $(x - 10 - 30) =$ the number of spaces the hour-hand moves over.

As the minute-hand moves 12 times as fast as the hour-hand,

$12(x - 10 - 30) =$ the number of spaces the minute-hand moves over.

$$\therefore 12(x - 10 - 30) = x$$
$$12x - 120 - 360 = x$$
$$11x = 480.$$
$$\therefore x = 43\tfrac{7}{11}.$$

Therefore the time is $43\frac{7}{11}$ minutes past 2 o'clock.

4. Find the time between 1 and 2 o'clock when the hands of a clock are at right angles to each other.

Let $x =$ the number of spaces the minute-hand moves over.

At one o'clock the minute-hand is 5 minute-spaces behind the hour-hand, and must be 15 minute-spaces ahead in order to be at right angles.

Then $(x - 5 - 15) =$ the number of spaces the hour-hand moves over.

As the minute-hand moves 12 times as fast as the hour-hand,

$12(x - 5 - 15) =$ the number of spaces the minute-hand moves over.

$$\therefore 12(x - 5 - 15) = x$$
$$12x - 60 - 180 = x$$
$$11x = 240$$
$$\therefore x = 21\tfrac{9}{11}.$$

Therefore the time is $21\frac{9}{11}$ minutes past 1 o'clock.

5. Find the time between 1 and 2 o'clock when the hands of a clock point in opposite directions.

Let $x =$ the number of spaces the minute-hand moves over.

At 1 o'clock the minute-hand is 5 minute-spaces behind the hour-hand, and must be 30 minute-spaces ahead for the hands to point in opposite directions.

Then $(x - 5 - 30) =$ the number of spaces the hour-hand moves over.

As the minute-hand moves 12 times as fast as the hour-hand,

$12(x - 5 - 30) =$ the number of spaces the minute-hand moves over.

$$\therefore 12(x - 5 - 30) = x$$
$$12x - 60 - 360 = x$$
$$11x = 420.$$
$$\therefore x = 38\tfrac{2}{11}.$$

Therefore the time is $38\frac{2}{11}$ minutes past 1 o'clock.

6. At what time between 7 and 8 o'clock are the hands of a watch together?

Let $x =$ the number of spaces the minute-hand moves over.

At 7 o'clock the minute-hand is 35 minute-spaces behind the hour-hand.

Then $(x - 35) =$ the number of spaces the hour-hand moves over.

As the minute-hand moves 12 times as fast as the hour-hand,

$12(x - 35) =$ the number of spaces the minute-hand moves over.

$$\therefore 12(x - 35) = x$$
$$12x - 420 = x$$
$$11x = 420.$$
$$\therefore x = 38\tfrac{2}{11}.$$

Therefore the time is $38\frac{2}{11}$ minutes past 7 o'clock.

Exercise 63. Page 117.

1. A rectangle has its length and breadth respectively 7 feet longer and 6 feet shorter than the side of the equivalent square. Find its area.

Let $x =$ the number of feet in side of square.
Then $x + 7 =$ the number of feet in length of rectangle.
and $x - 6 =$ the number of feet in breadth of rectangle.

$$\therefore (x + 7)(x - 6) = x^2$$
$$x^2 + x - 42 = x^2$$
$$x^2 - x^2 + x = 42.$$
$$\therefore x = 42.$$
$$\therefore x^2 = 1764.$$

Therefore the area is 1764 sq. ft.

2. The length of a floor exceeds its breadth by 5 feet. If each dimension were 1 foot more, the area of the floor would be 42 sq. ft. more. Find its dimensions.

Let $x =$ the number of feet in breadth.
Then $x + 5 =$ the number of feet in length,
$x(x + 5) =$ the number of square feet in area,
$x + 1 =$ the number of feet in breadth increased by 1 foot,
$x + 6 =$ the number of feet in length increased by 1 foot,
$(x + 1)(x + 6) =$ the number of square feet in area.

$$\therefore (x + 1)(x + 6) - x(x + 5) = 42$$
$$x^2 + 7x + 6 - x^2 - 5x = 42$$
$$7x - 5x = 42 - 6$$
$$2x = 36.$$
$$\therefore x = 18.$$

Therefore the dimensions are 18 feet and 23 feet.

3. A rectangle whose length is 6 feet more than its breadth would have its area 35 sq. ft. more, if each dimension were 1 foot more. Find its dimensions.

Let $x =$ the number of feet in breadth.
Then $x + 6 =$ the number of feet in length,
$x(x + 6) =$ the number of square feet in area,
$x + 1 =$ the number of feet in breadth increased by 1 foot,

and $\qquad x + 7 =$ the number of feet in length increased by 1 foot,

$(x + 1)(x + 7) =$ the number of square feet in area.

$$\therefore (x + 1)(x + 7) - x(x + 6) = 35$$
$$x^2 + 8x + 7 - x^2 - 6x = 35$$
$$8x - 6x = 35 - 7$$
$$2x = 28.$$
$$\therefore x = 14.$$

Therefore the dimensions are 14 feet and 20 feet.

4. The length of a rectangle exceeds its width by 3 feet. If the length is increased by 3 feet and the width diminished by 2 feet, the area will not be altered. Find its dimensions.

Let $\qquad x =$ the number of feet in width.

Then $\qquad x + 3 =$ the number of feet in length,

$x(x + 3) =$ the number of square feet in area,

$x + 6 =$ the number of feet in length increased 3 feet,

$x - 2 =$ the number of feet in width decreased 2 feet,

$(x - 2)(x + 6) =$ the number of square feet in area.

$$\therefore (x - 2)(x + 6) = x(x + 3)$$
$$x^2 + 4x - 12 = x^2 + 3x$$
$$x^2 - x^2 + 4x - 3x = 12.$$
$$\therefore x = 12.$$

Therefore the dimensions are 12 feet and 15 feet.

5. The length of a floor exceeds its width by 10 feet. If each dimension were 2 feet more, the area would be 144 sq. ft. more. Find its dimensions.

Let $\qquad x =$ the number of feet in width.

Then $\qquad x + 10 =$ the number of feet in length,

$x(x + 10) =$ the number of square feet in area,

$x + 2 =$ the number of feet in width increased by 2 feet,

and $\qquad x + 12 =$ the number of feet in length increased by 2 feet,

$(x + 2)(x + 12) =$ the number of square feet in area.

$$\therefore (x + 2)(x + 12) - x(x + 10) = 144$$
$$x^2 + 14x + 24 - x^2 - 10x = 144$$
$$14x - 10x = 144 - 24$$
$$4x = 120.$$
$$\therefore x = 30.$$

Therefore the dimensions are 30 feet and 40 feet.

Exercise 64. Page 121.

1. The sum of two angles is 120° 30′ 30″ and their difference 59° 30′ 30″. Find the angles.

One angle $= \dfrac{s+d}{2}$, and the other angle $= \dfrac{s-d}{2}$.

$$\begin{array}{rrr} 120^\circ & 30' & 30'' \\ 59 & 30 & 30 \\ \hline 180^\circ & 1' & 0'' \end{array} \qquad \begin{array}{rrr} 120^\circ & 30' & 30'' \\ 59 & 30 & 30 \\ \hline 61^\circ & 0' & 0'' \end{array}$$

$$\frac{180^\circ\ 1'}{2} = 90^\circ\ 0'\ 30''; \text{ and } \frac{61^\circ}{2} = 30^\circ\ 30'.$$

2. Find the interest of \$1000 for 3 years and 4 months at 4%.

$$i = prt. \qquad 3 \text{ yr. } 4 \text{ mo.} = 3\tfrac{1}{3} \text{ yr.}$$
$$i = \$1000 \times \tfrac{4}{100} \times 3\tfrac{1}{3} = \$133\tfrac{1}{3}.$$

3. Find the principal that will amount to \$2280 in 3 years and 6 months at 4%.

$$p = \frac{a}{1+rt}.$$
$$p = \frac{2280}{1 + (\frac{4}{100} \times \frac{7}{2})} = \frac{2280}{1.14} = 2000. \quad \$2000.$$

4. Find the principal that will produce \$280 interest in 2 years and 4 months at 3%.

$$p = \frac{i}{rt}.$$
$$p = \frac{280}{\frac{7}{3} \times \frac{3}{100}} = \frac{280}{0.07} = 4000. \quad \$4000.$$

5. Find the principal that will produce \$270 interest in 1 year and 6 months at 6%.

$$p = \frac{i}{rt}.$$
$$p = \frac{270}{\frac{6}{100} \times \frac{3}{2}} = \frac{270}{0.09} = 3000. \quad \$3000.$$

6. Find the principal that will amount to \$590 in 4 years at 4½%.

$$p = \frac{a}{1+rt}.$$
$$p = \frac{590}{1 + \left(\frac{4\frac{1}{2}}{100} \times 4\right)} = \frac{590}{1.18} = 500. \quad \$500.$$

7. Find the rate if the amount of \$250 for 4 years is \$300.

$$r = \frac{a-p}{pt}.$$

$$r = \frac{300-250}{250 \times \frac{4}{100}} = \frac{50}{10} = 5. \quad 5\%.$$

8. Find the rate if \$1000 amounts to \$2000 in 10 years and 8 months.

$$r = \frac{a-p}{pt}.$$

$$r = \frac{2000-1000}{1000 \times 16\frac{2}{3}} = \frac{1000}{1000 \times \frac{50}{3}} = \frac{3000}{50000} = \frac{6}{100}. \quad 6\%.$$

9. Find the time required for the interest on \$400 to be \$54 at $4\frac{1}{2}\%$.

$$i = ptr.$$

$$\therefore t = \frac{i}{pr}.$$

$$t = \frac{54}{400 \times \frac{4\frac{1}{2}}{100}} = \frac{54}{18} = 3. \quad 3 \text{ years.}$$

10. Find the time for \$160 to amount to \$250 at 6%.

$$t = \frac{a-p}{pr}.$$

$$t = \frac{250-160}{160 \times \frac{6}{100}} = \frac{90}{9.6} = 9\frac{3}{8} \text{ years.}$$

11. How much money must be invested at 5% to yield an annual income of \$1250?

$$p = \frac{i}{rt}.$$

$$p = \frac{1250}{0.05} = 25{,}000. \quad \$25{,}000.$$

12. Find the principal that will produce \$100 a month if invested at 6% per annum.

$$p = \frac{i}{rt}. \quad \$100 \text{ a month} = \$1200 \text{ a year.}$$

$$p = \frac{1200}{0.06} = 20{,}000. \quad \$20{,}000.$$

13. Find the rate if the interest on \$1000 for 8 months is \$40.

$r = \frac{i}{pt}$. \$40 for 8 months = \$60 for a year.

$r = \frac{60}{1000} = \frac{6}{100}$. 6%.

14 Find the time for a sum of money on interest at 5% to double itself.

$t = \frac{i}{pr}$. Suppose $p = \$100$. Then $i = \$100$.

$t = \frac{100}{5} = 20$. 20 years.

Exercise 65. Page 124.

Solve by addition or subtraction :

1. $\left.\begin{aligned} 5x + 4y &= 14 \\ 17x - 3y &= 31 \end{aligned}\right\}$

$$\begin{aligned} 5x + 4y &= 14 \qquad (1) \\ 17x - 3y &= 31 \qquad (2) \end{aligned}$$

Multiply (1) by 3, and (2) by 4.

$$\begin{aligned} 15x + 12y &= 42 \qquad (3) \\ 68x - 12y &= 124 \qquad (4) \end{aligned}$$

Add, $83x = 166$

$$\therefore x = 2$$

Substitute value of x in (1).

$$\begin{aligned} 10 + 4y &= 14 \\ 4y &= 4 \\ \therefore y &= 1 \end{aligned}$$

2. $\left.\begin{aligned} 3x - 2y &= 5 \\ 2x + 5y &= 16 \end{aligned}\right\}$

$$\begin{aligned} 3x - 2y &= 5 \qquad (1) \\ 2x + 5y &= 16 \qquad (2) \end{aligned}$$

Multiply (1) by 2, and (2) by 3.

$$\begin{aligned} 6x - 4y &= 10 \qquad (3) \\ 6x + 15y &= 48 \qquad (4) \end{aligned}$$

Subtract, $-19y = -38$

$$\therefore y = 2$$

Substitute value of y in (2).

$$\begin{aligned} 2x + 10 &= 16 \\ 2x &= 16 - 10 \\ 2x &= 6 \\ \therefore x &= 3 \end{aligned}$$

3. $\left.\begin{aligned} 2x - 3y &= 7 \\ 5x + 2y &= 27 \end{aligned}\right\}$

$$\begin{aligned} 2x - 3y &= 7 \qquad (1) \\ 5x + 2y &= 27 \qquad (2) \end{aligned}$$

Multiply (1) by 2, and (2) by 3.

$$\begin{aligned} 4x - 6y &= 14 \\ 15x + 6y &= 81 \end{aligned}$$

Add, $19x = 95$

$$\therefore x = 5$$

Substitute value of x in (1).

$$10 - 3y = 7$$
$$-3y = -3$$
$$\therefore y = 1$$

4. $\left.\begin{array}{l} 7x + 6y = 20 \\ 2x + 5y = 9 \end{array}\right\}$

$$\begin{array}{rll} 7x + 6y = 20 & & (1) \\ 2x + 5y = 9 & & (2) \end{array}$$

Multiply (1) by 2, and (2) by 7.

$$\begin{array}{rll} 14x + 12y = 40 & & (3) \\ 14x + 35y = 63 & & (4) \end{array}$$

Subtract, $-23y = -23$

$$\therefore y = 1$$

Substitute value of y in (2).

$$2x + 5 = 9$$
$$2x = 4$$
$$\therefore x = 2$$

5. $\left.\begin{array}{l} x + 5y = 11 \\ 3x + 2y = 7 \end{array}\right\}$

$$\begin{array}{rll} x + 5y = 11 & & (1) \\ 3x + 2y = 7 & & (2) \end{array}$$

Multiply (1) by 3.

$$3x + 15y = 33 \qquad (3)$$

(2) is $3x + 2y = 7$

Subtract, $13y = 26$

$$\therefore y = 2$$

Substitute value of y in (1).

$$x + 10 = 11$$
$$\therefore x = 1$$

6. $\left.\begin{array}{l} 3x - 5y = 13 \\ 4x - 7y = 17 \end{array}\right\}$

$$\begin{array}{rll} 3x - 5y = 13 & & (1) \\ 4x - 7y = 17 & & (2) \end{array}$$

Multiply (1) by 4, and (2) by 3.

$$\begin{array}{rll} 12x - 20y = 52 & & (3) \\ 12x - 21y = 51 & & (4) \end{array}$$

Subtract, $y = 1$

Substitute value of y in (1).

$$3x - 5 = 13$$
$$3x = 18$$
$$\therefore x = 6$$

7. $\left.\begin{array}{l} 8x - y = 3 \\ 7x + 2y = 63 \end{array}\right\}$

$$\begin{array}{rll} 8x - y = 3 & & (1) \\ 7x + 2y = 63 & & (2) \end{array}$$

Multiply (1) by 2.

$$16x - 2y = 6$$

(2) is $7x + 2y = 63$

Add, $23x = 69$

$$\therefore x = 3$$

Substitute value of x in (1).

$$24 - y = 3$$
$$-y = -21$$
$$\therefore y = 21$$

8. $\left.\begin{array}{l} 5x - 4y = 7 \\ 7x + 3y = 70 \end{array}\right\}$

$$\begin{array}{rll} 5x - 4y = 7 & & (1) \\ 7x + 3y = 70 & & (2) \end{array}$$

Multiply (1) by 3, and (2) by 4.

$$15x - 12y = 21$$
$$28x + 12y = 280$$

Add, $43x = 301$

$$\therefore x = 7$$

Substitute value of x in (1).

$$35 - 4y = 7$$
$$-4y = 7 - 35$$
$$4y = 28$$
$$\therefore y = 7$$

9. $\left.\begin{aligned} x + 21y &= 2 \\ 2x + 27y &= 19 \end{aligned}\right\}$

$$\begin{aligned} x + 21y &= 2 && (1) \\ 2x + 27y &= 19 && (2) \end{aligned}$$

Multiply (1) by 2.

$$\begin{aligned} 2x + 42y &= 4 && (3) \\ \text{(2) is} \quad 2x + 27y &= 19 && (4) \\ \text{Subtract,} \quad 15y &= -15 \\ \therefore y &= -1 \end{aligned}$$

Substitute value of y in (1).

$$\begin{aligned} x - 21 &= 2 \\ \therefore x &= 23 \end{aligned}$$

10. $\left.\begin{aligned} 6x - 13y &= -1 \\ 5x - 12y &= -2 \end{aligned}\right\}$

$$\begin{aligned} 6x - 13y &= -1 && (1) \\ 5x - 12y &= -2 && (2) \end{aligned}$$

Multiply (1) by 5, and (2) by 6.

$$\begin{aligned} 30x - 65y &= -5 && (3) \\ 30x - 72y &= -12 && (4) \\ \text{Subtract,} \quad 7y &= 7 \\ \therefore y &= 1 \end{aligned}$$

Substitute value of y in (2).

$$\begin{aligned} 5x - 12 &= -2 \\ 5x &= 10 \\ \therefore x &= 2 \end{aligned}$$

11. $\left.\begin{aligned} 7x + y &= 265 \\ 3x - 5y &= 5 \end{aligned}\right\}$

$$\begin{aligned} 7x + y &= 265 && (1) \\ 3x - 5y &= 5 && (2) \end{aligned}$$

Multiply (1) by 3, and (2) by 7.

$$\begin{aligned} 21x + 3y &= 795 && (3) \\ 21x - 35y &= 35 && (4) \\ \text{Subtract,} \quad 38y &= 760 \\ \therefore y &= 20 \end{aligned}$$

Substitute value of y in (1).

$$\begin{aligned} 7x + 20 &= 265 \\ 7x &= 245 \\ \therefore x &= 35 \end{aligned}$$

12. $\left.\begin{aligned} 2x + 3y &= 7 \\ 8x - 5y &= 11 \end{aligned}\right\}$

$$\begin{aligned} 2x + 3y &= 7 && (1) \\ 8x - 5y &= 11 && (2) \end{aligned}$$

Multiply (1) by 5, and (2) by 3.

$$\begin{aligned} 10x + 15y &= 35 \\ 24x - 15y &= 33 \\ \text{Add,} \quad 34x &= 68 \\ \therefore x &= 2 \end{aligned}$$

Substitute value of x in (1).

$$\begin{aligned} 4 + 3y &= 7 \\ 3y &= 3 \\ \therefore y &= 1 \end{aligned}$$

13. $\left.\begin{aligned} 5x + 7y &= 19 \\ 7x + 4y &= 15 \end{aligned}\right\}$

$$\begin{aligned} 5x + 7y &= 19 && (1) \\ 7x + 4y &= 15 && (2) \end{aligned}$$

Multiply (1) by 4, and (2) by 7.

$$\begin{aligned} 20x + 28y &= 76 && (3) \\ 49x + 28y &= 105 && (4) \\ \text{Subt.,} \quad -29x &= -29 \\ \therefore x &= 1 \end{aligned}$$

Substitute value of x in (1).

$$\begin{aligned} 5 + 7y &= 19 \\ 7y &= 14 \\ \therefore y &= 2 \end{aligned}$$

14. $\left.\begin{aligned} 11x - 12y &= 9 \\ 4x + 5y &= 22 \end{aligned}\right\}$

$$\begin{aligned} 11x - 12y &= 9 && (1) \\ 4x + 5y &= 22 && (2) \end{aligned}$$

Multiply (1) by 4, and (2) by 11.

$$\begin{aligned} 44x - 48y &= 36 && (3) \\ 44x + 55y &= 242 && (4) \\ \text{Subtract,} \quad -103y &= -206 \\ \therefore y &= 2 \end{aligned}$$

Substitute value of y in (2).

$$\begin{aligned} 4x + 10 &= 22 \\ 4x &= 12 \\ \therefore x &= 3 \end{aligned}$$

15. $\left.\begin{aligned} x + 8y &= 17 \\ 7x - 3y &= 1 \end{aligned}\right\}$

$$x + 8y = 17 \quad (1)$$
$$7x - 3y = 1 \quad (2)$$

Multiply (1) by 3, and (2) by 8.

$$3x + 24y = 51$$
$$56x - 24y = 8$$

Add, $59x = 59$

$$\therefore x = 1$$

Substitute value of x in (1).

$$1 + 8y = 17$$
$$8y = 16$$
$$\therefore y = 2$$

16. $\left.\begin{aligned} 4x + 3y &= 25 \\ 5x - 4x &= 8 \end{aligned}\right\}$

$$4x + 3y = 25 \quad (1)$$
$$5x - 4y = 8 \quad (2)$$

Multiply (1) by (4), and (2) by (3).

$$16x + 12y = 100$$
$$15x - 12y = 24$$

Add, $31x = 124$

$$\therefore x = 4$$

Substitute value of x in (1).

$$16 + 3y = 25$$
$$3y = 9$$
$$\therefore y = 3$$

Clear of fractions and solve:

17. $\left.\begin{aligned} \frac{2x}{3} - \frac{5y}{4} &= 3 \\ \frac{7x}{4} - \frac{5y}{3} &= \frac{43}{3} \end{aligned}\right\}$

$$\frac{2x}{3} - \frac{5y}{4} = 3 \quad (1)$$
$$\frac{7x}{4} - \frac{5y}{3} = \frac{43}{3} \quad (2)$$

Multiply both equations by 12.

$$8x - 15y = 36 \quad (3)$$
$$21x - 20y = 172 \quad (4)$$

Multiply (3) by 4, and (4) by 3.

$$32x - 60y = 144 \quad (5)$$
$$63x - 60y = 516 \quad (6)$$

Subt., $-31x = -372$

$$\therefore x = 12$$

Substitute value of x in (1).

$$8 - \frac{5y}{4} = 3$$
$$-\frac{5y}{4} = -5$$
$$5y = 20$$
$$\therefore y = 4$$

18. $\left.\begin{aligned} \frac{7x}{6} + \frac{6y}{7} &= 32 \\ \frac{5x}{4} - \frac{2y}{3} &= 1 \end{aligned}\right\}$

$$\frac{7x}{6} + \frac{6y}{7} = 32 \quad (1)$$
$$\frac{5x}{4} - \frac{2y}{3} = 1 \quad (2)$$

Multiply (1) by 42, and (2) by 12.

$$49x + 36y = 1344 \quad (3)$$
$$15x - 8y = 12 \quad (4)$$

Multiply (3) by 2, and (4) by 9.

$$98x + 72y = 2688 \quad (5)$$
$$135x - 72y = 108 \quad (6)$$

Add, $233x = 2796$

$$\therefore x = 12$$

Substitute value of x in (2).

$$15 - \frac{2y}{3} = 1$$
$$-\frac{2y}{3} = -14$$
$$\therefore y = 21$$

19. $$\left.\begin{array}{r}\frac{x+y}{4}-\frac{7x-5y}{11}=3\\ \frac{x}{5}-\frac{2y}{7}+1=0\end{array}\right\}$$

$$\frac{x+y}{4}-\frac{7x-5y}{11}=3 \qquad (1)$$

$$\frac{x}{5}-\frac{2y}{7}+1=0 \qquad (2)$$

Multiply (1) by 44, and (2) by 35.

$$11x+11y-28x+20y=132$$

that is,

$$-17x+31y=132 \qquad (3)$$

$$7x-10y=-35 \qquad (4)$$

Multiply (3) by 7, and (4) by 17.

$$-119x+217y=\ \ 924 \qquad (5)$$

$$119x-170y=-595 \qquad (6)$$

Add,

$$47y=\ \ 329$$

$$\therefore y=7$$

Substitute value of y in (2).

$$\frac{x}{5}-2+1=0$$

$$\frac{x}{5}=1$$

$$\therefore x=5$$

20. $$\left.\begin{array}{r}\frac{6x+7y}{2}=22\\ \frac{55y-2x}{5}=20\end{array}\right\}$$

$$\frac{6x+7y}{2}=22 \qquad (1)$$

$$\frac{55y-2x}{5}=20 \qquad (2)$$

Multiply (1) by 2, and (2) by 5.

$$6x+7y=44 \qquad (3)$$

$$55y-2x=100 \qquad (4)$$

Multiply (4) by 3, $\quad -6x+165y=300$

(3) is, $\quad 6x+\ \ 7y=\ \ 44$

Add, $\quad 172y=344$

$$\therefore y=2$$

Substitute value of y in (3).

$$6x+14=44$$

$$6x=30$$

$$\therefore x=5$$

21. $\left.\begin{aligned}\frac{x+y}{2}-\frac{x-y}{3}&=8\\ \frac{x+y}{3}+\frac{x-y}{4}&=11\end{aligned}\right\}$

$$\frac{x+y}{2}-\frac{x-y}{3}=8 \qquad (1)$$

$$\frac{x+y}{3}+\frac{x-y}{4}=11 \qquad (2)$$

Multiply (1) by 6, and (2) by 12.

$$3x+3y-2x+2y=48$$

that is, $$x+5y=48 \qquad (3)$$

$$4x+4y+3x-3y=132$$

that is, $$7x+y=132 \qquad (4)$$

Multiply (3) by 7, $$7x+35y=336 \qquad (5)$$

Subtract (5) from (4), $$-34y=-204$$

$$\therefore y=6$$

Substitute value of y in (3), $x+30=48$

$$\therefore x=18$$

22. $\left.\begin{aligned}\frac{8x-5y}{7}+\frac{11y-4x}{5}&=4\\ \frac{17x-13y}{5}+\frac{2x}{3}&=7\end{aligned}\right\}$

$$\frac{8x-5y}{7}+\frac{11y-4x}{5}=4 \qquad (1)$$

$$\frac{17x-13y}{5}+\frac{2x}{3}=7 \qquad (2)$$

Multiply (1) by 35, and (2) by 15.

$$40x-25y+77y-28x=140$$

or, $$12x+52y=140 \qquad (3)$$

$$51x-39y+10x=105$$

or, $$61x-39y=105 \qquad (4)$$

Divide (3) by 4, and multiply the quotient by 3.

$$9x+39y=105 \qquad (5)$$

(4) is, $$61x-39y=105$$

Add, $$70x=210$$

$$\therefore x=3$$

Substitute value of x in (5).

$$27+39y=105$$

$$39y=78$$

$$\therefore y=2$$

23. $$\left.\begin{aligned}\frac{5x-3y}{3}+\frac{7x-5y}{11}=4\\ \frac{15y-3x}{7}+\frac{7y-3x}{5}=4\end{aligned}\right\}$$

$$\frac{5x-3y}{3}+\frac{7x-5y}{11}=4 \qquad (1)$$

$$\frac{15y-3x}{7}+\frac{7y-3x}{5}=4 \qquad (2)$$

Multiply (1) by 33, and (2) by 35.

$$55x-33y+21x-15y=132$$

or, $$76x-48y=132$$

or, dividing by 4, $$19x-12y=33 \qquad (3)$$

$$75y-15x+49y-21x=140$$

or, $$-36x+124y=140$$

or, dividing by 4, $$-9x+31y=35 \qquad (4)$$

Multiply (3) by 9, and (4) by 19.

$$171x-108y=297$$
$$-171x+589y=665$$

Add, $$481y=962$$
$$\therefore y=2$$

Substitute value of y in (3).

$$19x-24=33$$
$$19x=33+24$$
$$19x=57$$
$$\therefore x=3$$

24. $$\left.\begin{aligned}\frac{2x-3}{4}-\frac{y-8}{5}=\frac{y+3}{4}\\ \frac{x-7}{3}+\frac{4y+1}{11}=3\end{aligned}\right\}$$

$$\frac{2x-3}{4}-\frac{y-8}{5}=\frac{y+3}{4} \qquad (1)$$

$$\frac{x-7}{3}+\frac{4y+1}{11}=3 \qquad (2)$$

Multiply (1) by 20, and (2) by 33.

$$10x-15-4y+32=5y+15$$

or, $$10x-9y=-2 \qquad (3)$$

$$11x-77+12y+3=99$$

or, $$11x+12y=173 \qquad (4)$$

Multiply (3) by 11, and (4) by 10.

$$110x - 99y = -22 \quad (5)$$
$$110x + 120y = 1730 \quad (6)$$

Subtract,

$$-219y = -1752$$
$$\therefore y = 8$$

Substitute value of y in (3).

$$10x - 72 = -2$$
$$\therefore x = 7$$

25. $$\left.\begin{aligned}\frac{x-2y}{6} - \frac{x+3y}{4} &= \frac{3}{2}\\ \frac{2x-y}{6} - \frac{3x+y}{4} &= \frac{5y}{4}\end{aligned}\right\}$$

$$\frac{x-2y}{6} - \frac{x+3y}{4} = \frac{3}{2} \quad (1)$$
$$\frac{2x-y}{6} - \frac{3x+y}{4} = \frac{5y}{4} \quad (2)$$

Multiply (1) by 12, and (2) by 12.

$$2x - 4y - 3x - 9y = 18$$

or,

$$-x - 13y = 18 \quad (3)$$
$$4x - 2y - 9x - 3y = 15y$$

or,

$$-5x - 20y = 0$$

or,

$$-x - 4y = 0 \quad (4)$$

(3) is

$$-x - 13y = 18$$

Subtract,

$$9y = -18$$
$$\therefore y = -2$$

Substitute value of y in (4).

$$-x + 8 = 0$$
$$\therefore x = 8$$

26. $$\left.\begin{aligned}\frac{x}{a+b} + \frac{y}{a-b} &= \frac{1}{a-b}\\ \frac{x}{a+b} - \frac{y}{a-b} &= \frac{1}{a+b}\end{aligned}\right\}$$

$$\frac{x}{a+b} + \frac{y}{a-b} = \frac{1}{a-b} \quad (1)$$
$$\frac{x}{a+b} - \frac{y}{a-b} = \frac{1}{a+b} \quad (2)$$

Add,

$$\frac{2x}{a+b} = \frac{1}{a-b} + \frac{1}{a+b} = \frac{2a}{(a+b)(a-b)}$$

Multiply by $a+b$, $$2x=\frac{2a}{a-b}$$

Divide by 2, $$x=\frac{a}{a-b}$$

Subtract, $$\frac{2y}{a-b}=\frac{1}{a-b}-\frac{1}{a+b}=\frac{2b}{(a+b)(a-b)}$$

Multiply by $a-b$, $$2y=\frac{2b}{a+b}$$

Divide by 2, $$y=\frac{b}{a+b}$$

Exercise 66. Page 127.

1. If A gives B $200, A will then have half as much money as B; but if B gives A $200, B will have one-third as much as A. How much has each?

Let $x=$ the number of dollars A has,

and $y=$ the number of dollars B has.

Then, after A gives B $200,

$x-200=$ the number of dollars A has,

$y+200=$ the number of dollars B has.

Since A's money is now one-half of B's, we have,

$$x-200=\tfrac{1}{2}(y+200) \qquad (1)$$

If B gives A $200,

$x+200=$ the number of dollars A has,

$y-200=$ the number of dollars B has.

Since B's money is one-third of A's, we have,

$$y-200=\tfrac{1}{3}(x+200) \qquad (2)$$

Clear (1) and (2) of fractions.

$$2x-400=y+200$$

that is, $$2x-y=600 \qquad (3)$$

$$3y-600=x+200$$

that is, $$x-3y=-800 \qquad (4)$$

Multiply (3) by 3, $$6x-3y=1800 \qquad (5)$$

Subtract (4) from (5), $$5x=2600$$

$$\therefore x=520$$

Substitute value of x in (1).

$$520-200=\tfrac{1}{2}y+100$$

$$\tfrac{1}{2}y=220$$

$$y=440$$

A, $520; B, $440.

2. Half the sum of two numbers is 20, and three times their difference is 18. Find the numbers.

Let $x =$ the greater number,
and $y =$ the smaller number.

Then $$\frac{x+y}{2} = 20 \qquad (1)$$

and $$3(x-y) = 18 \qquad (2)$$

Multiply (1) by 2, and divide (2) by 3.

$$x + y = 40 \qquad (3)$$
$$x - y = 6 \qquad (4)$$

Add, $$2x = 46$$
$$\therefore x = 23$$

Subtract (4) from (3), $$2y = 34$$
$$\therefore y = 17$$

23 and 17.

3. The sum of two numbers is 36, and their difference is equal to one-eighth of the smaller number increased by 2. Find the numbers.

Let $x =$ the greater number,
and $y =$ the smaller number.

Then $$x + y = 36 \qquad (1)$$

and $$x - y = \frac{y}{8} + 2 \qquad (2)$$

Multiply (2) by 8, $$8x - 8y = y + 16$$

that is, $$8x - 9y = 16 \qquad (3)$$

Multiply (1) by 8, $$8x + 8y = 288 \qquad (4)$$

Subtract (3) from (4), $$17y = 272$$
$$\therefore y = 16$$

Substitute value of y in (1),
$$x + 16 = 36$$
$$\therefore x = 20$$

20 and 16.

4. If 4 yards of velvet and 3 yards of silk are sold for $33, and 5 yards of velvet and 6 yards of silk for $48, what is the price per yard of the velvet and of the silk?

Let $x =$ the number of dollars the velvet costs a yard,
and $y =$ the number of dollars the silk costs a yard.

Then	$4x + 3y = 33$	(1)
and	$5x + 6y = 48$	(2)
Multiply (1) by 2,	$8x + 6y = 66$	(3)
Subtract (2) from (3),	$3x = 18$	
	$\therefore x = 6$	

Substitute value of x in (1).

$$24 + 3y = 33$$
$$3y = 9$$
$$\therefore y = 3 \qquad \text{Velvet, \$6; silk, \$3.}$$

5. If 7 bushels of wheat and 10 of rye are sold for $15, and 4 bushels of wheat and 5 of rye are sold for $8, what is the price per bushel of the wheat and of the rye?

Let $x =$ the number of dollars the wheat costs a bushel,
and $y =$ the number of dollars the rye costs a bushel.

Then	$7x + 10y = 15$	(1)
and	$4x + 5y = 8$	(2)
Multiply (2) by 2,	$8x + 10y = 16$	(3)
(1) is	$7x + 10y = 15$	(1)
Subtract (1) from (3),	$x = 1$	
Substitute value of x in (2),	$4 + 5y = 8$	
	$5y = 4$	
	$\therefore y = \frac{4}{5}$ Wheat, \$1; rye, \$$\frac{4}{5}$.	

6. If 12 pounds of tea and 4 pounds of coffee cost $7, and 4 pounds of tea and 12 pounds of coffee cost $5, what is the price per pound of tea and of coffee?

Let $x =$ the number of dollars the tea costs a pound,
and $y =$ the number of dollars the coffee costs a pound.

Then	$12x + 4y = 7$	(1)
and	$4x + 12y = 5$	(2)
Multiply (2) by 3,	$12x + 36y = 15$	(3)
(1) is	$12x + 4y = 7$	(1)
Subtract (1) from 3,	$32y = 8$	
	$\therefore y = \frac{1}{4}$	

Substitute value of y in (2).

$$4x + 3 = 5$$
$$4x = 2$$
$$\therefore x = \frac{1}{2} \qquad \text{Tea, \$}\tfrac{1}{2}\text{; coffee, \$}\tfrac{1}{4}.$$

7. Six horses and 7 cows can be bought for \$1000, and 11 horses and 13 cows for \$1844. Find the value of a horse and of a cow.

Let $x =$ the number of dollars a horse costs,
and $y =$ the number of dollars a cow costs.

Then
$$6x + 7y = 1000 \qquad (1)$$
and
$$11x + 13y = 1844 \qquad (2)$$
Multiply (1) by 11, and (2) by 6.
$$66x + 77y = 11,000 \qquad (3)$$
$$66x + 78y = 11,064 \qquad (4)$$
Subtract (3) from (4), $y = 64$

Substitute value of y in (1).
$$6x + 448 = 1000$$
$$6x = 552$$
$$\therefore x = 92$$
Horse, \$92; cow, \$64.

Exercise 67. Page 128.

1. If the numerator of a certain fraction is increased by 2 and its denominator diminished by 2, its value will be 1. If the numerator is increased by the denominator and the denominator is diminished by 5, its value will be 5. Find the fraction.

Let $\frac{x}{y} =$ the required fraction.

Then
$$\frac{x+2}{y-2} = 1 \qquad (1)$$
and
$$\frac{x+y}{y-5} = 5 \qquad (2)$$
Clear of fractions. $x + 2 = y - 2$
that is, $x - y = -4 \qquad (3)$
$$x + y = 5y - 25$$
that is, $x - 4y = -25 \qquad (4)$
(3) is $x - y = -4$
Subtract (4) from (3) $3y = 21$
$$\therefore y = 7$$
Substitute value of y in (3).
$$x = 3$$
Therefore the required fraction is $\frac{3}{7}$.

2. If 1 is added to the denominator of a fraction, its value will be $\frac{1}{2}$. If 2 is added to its numerator, its value will by $\frac{3}{5}$. Find the fraction.

Let $\frac{x}{y}$ = the required fraction.

Then
$$\frac{x}{y+1}=\frac{1}{2} \qquad (1)$$
and
$$\frac{x+2}{y}=\frac{3}{5} \qquad (2)$$

Clear of fractions, $2x=y+1$

that is, $2x-y=1 \qquad (3)$

$5x+10=3y$

that is, $5x-3y=-10 \qquad (4)$

Multiply (3) by 3, $6x-3y=3 \qquad (5)$

Subtract (4) from (5), $x = 13$

Substitute value of x in (3).

$$26-y=1$$
$$\therefore y=25$$

Therefore the required fraction is $\frac{13}{25}$.

3. If 1 is added to the numerator of a fraction, its value will be $\frac{1}{5}$. If 1 is added to its denominator, its value will be $\frac{1}{7}$. Find the fraction.

Let $\frac{x}{y}$ = the required fraction.

Then
$$\frac{x+1}{y}=\frac{1}{5} \qquad (1)$$
and
$$\frac{x}{y+1}=\frac{1}{7} \qquad (2)$$

Clear of fractions, $5x+5=y$

that is, $5x-y=-5 \qquad (3)$

$7x=y+1$

that is, $7x-y=1 \qquad (4)$

(3) is $5x-y=-5$

Subtract (3) from (4), $2x = 6$

$$\therefore x=3$$

Substitute value of x in (3).

$$15-y=-5$$
$$\therefore y=20$$

Therefore the required fraction is $\frac{3}{20}$.

4 If the numerator of a fraction is doubled and its denominator diminished by 1, its value will be $\frac{1}{2}$. If its denominator is doubled and its numerator increased by 1, its value will be $\frac{1}{7}$. Find the fraction.

Let $\frac{x}{y} =$ the required fraction.

Then, $$\frac{2x}{y-1} = \frac{1}{2} \qquad (1)$$

and $$\frac{x+1}{2y} = \frac{1}{7} \qquad (2)$$

Clear of fractions, $4x = y - 1$

that is, $4x - y = -1 \qquad (3)$

$7x + 7 = 2y$

that is $7x - 2y = -7 \qquad (4)$

Multiply (3) by 2 $8x - 2y = -2 \qquad (5)$

Subtract (4) from (5), $x = 5$

Substitute value of x in (3), $20 - y = -1$

$\therefore y = 21$

Therefore the required fraction is $\frac{5}{21}$.

5. In a certain proper fraction the difference between the numerator and the denominator is 15. If the numerator is multiplied by 4 and the denominator increased by 6, its value will be 1. Find the fraction.

Let $\frac{x}{y} =$ the required fraction.

Then $y - x = 15 \qquad (1)$

and $$\frac{4x}{y+6} = 1 \qquad (2)$$

Clear (2) of fractions, $4x = y + 6$

that is, $4x - y = 6 \qquad (3)$

(1) is $-x + y = 15$

Add, $3x = 21$

$\therefore x = 7$

Substitute value of x in (1), $y - 7 = 15$

$\therefore y = 22.$

Therefore the required fraction is $\frac{7}{22}$.

Exercise 68. Page 129.

1. The sum of the two digits of a number is 9, and if 9 is added to the number, the digits will be reversed. Find the number.

Let $x =$ the tens' digit,
and $y =$ the units' digit.
Then $10x + y =$ the number.
Hence $x + y = 9$ (1)
and $10x + y + 9 = 10y + x$ (2)
From (2), $9x - 9y = -9$
that is, dividing by 9 $x - y = -1$ (3)
(1) is $x + y = 9$
Add, $2x = 8$
$\therefore x = 4$
Subtract (3) from (1), $2y = 10$
$\therefore y = 5$

Therefore the number is 45.

2. A certain number of two digits is equal to eight times the sum of its digits, and if 45 is subtracted from the number, the digits will be reversed. Find the number.

Let $x =$ the tens' digit,
and $y =$ the units' digit.
Then $10x + y =$ the number.
Hence $10x + y = 8(x + y)$ (1)
and $10x + y - 45 = 10y + x$ (2)
From (1), $10x + y = 8x + 8y$
that is, $2x - 7y = 0$ (3)
From (2), $9x - 9y = 45$
that is, dividing by 9, $x - y = 5$ (4)
Multiply (4) by 2, $2x - 2y = 10$ (5)
(3) is $2x - 7y = 0$
Subtract (3) from (5), $5y = 10$
$\therefore y = 2$
Substitute value of y in (4), $x - 2 = 5$
$\therefore x = 7$

Therefore the number is 72.

3. The sum of a certain number of two digits and of the number formed by reversing the digits is 132, and the difference of these numbers is 18. Find the numbers.

Let $x =$ the tens' digit
and $y =$ the units' digit.
Then $10x + y =$ the number.

$$
\begin{array}{lrl}
\text{Hence} & 10x + y + 10y + x = 132 & (1) \\
\text{and} & (10x + y) - (10y + x) = 18 & (2) \\
\text{From (1),} & 11x + 11y = 132 & \\
\text{that is,} & x + y = 22 & (3) \\
\text{From (2),} & 9x - 9y = 18 & \\
\text{that is,} & x - y = 2 & (4) \\
\text{(3) is} & x + y = 12 & \\
\text{Subtract (4) from (3),} & 2y = 10 & \\
 & \therefore y = 5 & \\
\text{Add (3) and (4),} & 2x = 14 & \\
 & \therefore x = 7 &
\end{array}
$$

Therefore the numbers are 75 and 57.

4. The sum of the two digits of a number is 9, and if the number is divided by the sum of its digits, the quotient is 6. Find the number.

Let $x =$ the tens' digit,
and $y =$ the units' digit.
Then $10x + y =$ the number.

$$
\begin{array}{lrl}
\text{Hence} & x + y = 9 & (1) \\
\text{and} & \dfrac{10x + y}{x + y} = 6 & (2)
\end{array}
$$

Clear (2) of fractions.

$$
\begin{array}{lrl}
 & 10x + y = 6x + 6y & \\
\text{that is,} & 4x - 5y = 0 & (3) \\
\text{Multiply (1) by 4,} & 4x + 4y = 36 & (4) \\
\text{Subtract (3) from (4),} & 9y = 36 & \\
 & \therefore y = 4 &
\end{array}
$$

Substitute value of y in (1).

$$
\begin{array}{r}
x + 4 = 9 \\
\therefore x = 5
\end{array}
$$

Therefore the number is 54.

Exercise 69. Page 130.

1. A sum of money, at simple interest, amounted in 5 years to \$3000, and in 6 years to \$3100. Find the sum and the rate of interest.

Let $x =$ the principal,
and $y =$ the rate of interest.

Then $$x + \frac{5xy}{100} = 3000 \quad (1)$$

and $$x + \frac{6xy}{100} = 3100 \quad (2)$$

Clear of fractions, $$100x + 5xy = 300{,}000 \quad (3)$$
$$100x + 6xy = 310{,}000 \quad (4)$$

Multiply (3) by 6, and (4) by 5.
$$600x + 30xy = 1{,}800{,}000$$
$$500x + 30xy = 1{,}550{,}000$$

Subtract, $$100x = 250{,}000$$
$$\therefore x = 2500$$

Substitute value of x in (3).
$$250{,}000 + 12{,}500y = 300{,}000$$
$$125y = 3000 - 2500$$
$$125y = 500$$
$$\therefore y = 4$$

\$2500 at 4%.

2. A sum of money, at simple interest, amounted in 10 months to \$1680, and in 18 months to \$1744. Find the sum and the rate of interest.

Let $x =$ the principal.
and $y =$ the rate of interest per month.

Then $$x + \frac{10xy}{100} = 1680 \quad (1)$$

and $$x + \frac{18xy}{100} = 1744 \quad (2)$$

Clear of fractions, $$100x + 10xy = 168{,}000$$
that is, $$10x + xy = 16{,}800$$
$$100x + 18xy = 174{,}400$$
that is, $$50x + 9xy = 87{,}200 \quad (4)$$
Multiply (3) by 9, $$90x + 9xy = 151{,}200 \quad (5)$$
Subtract (4) from (5), $$40x = 64{,}000$$
$$\therefore x = 1600$$

Substitute value of x in (3).

$$16{,}000 + 1600y = 16{,}800$$

Transpose, and divide by 100, $16y = 168 - 160$

$$16y = 8$$

$$\therefore y = \tfrac{1}{2}$$

$$\tfrac{1}{2}\% \text{ a month} = 6\% \text{ a year.}$$

\$1600 at 6%.

3. A man has \$10,000 invested, a part at 4%, and the remainder at 5%. The annual income from his 4% investment is \$40 dollars more than from his 5% investment. Find the sum invested at 4% and at 5%.

Let $x =$ the number of dollars invested at 4%,

and $y =$ the number of dollars invested at 5%.

Then $x + y = 10{,}000$ (1)

$\frac{4x}{100} =$ the number of dollars income from the 4%,

and $\frac{5y}{100} =$ the number of dollars income from the 5%.

Then $$\frac{4x}{100} - \frac{5y}{100} = 40 \qquad (2)$$

Clear (2) of fractions, $4x - 5y = 4{,}000$ (3)

Multiply (1) by 4, $4x + 4y = 40{,}000$ (4)

Subtract (3) from (4), $9y = 36{,}000$

$$\therefore y = 4000$$

Substitute value of y in (1).

$$x + 4000 = 10{,}000$$

$$\therefore x = 6000$$

\$6000 at 4%; \$4000 at 5%.

Exercise 70. Page 131.

Miscellaneous Examples.

1. Half the sum of two numbers is 20; and 5 times their difference is 20. Find the numbers.

Let $x =$ the greater number,

and $y =$ the smaller number.

Then $$\frac{x+y}{2} = 20 \qquad (1)$$

and $$5(x - y) = 20 \qquad (2)$$

Multiply (1) by 2, and divide (2) by 5.

$$x + y = 40 \qquad (3)$$

$$x - y = 4 \qquad (4)$$

Add, $2x = 44$

$$\therefore x = 22$$

Subtract (4) from (3), $2y = 36$

$$\therefore y = 18$$

Therefore the numbers are 22 and 18.

2. A certain number when divided by a second number gives 7 for a quotient and 4 for a remainder. If three times the first number is divided by twice the second number, the quotient is 11 and the remainder 4. Find the numbers.

Let $x =$ one number,

and $y =$ the second number.

Then
$$\frac{x-4}{y} = 7 \qquad (1)$$

and
$$\frac{3x-4}{2y} = 11 \qquad (2)$$

Clear (1) of fractions, $x - 4 = 7y$

that is, $x - 7y = 4 \qquad (3)$

Clear (2) of fractions, $3x - 4 = 22y$

that is, $3x - 22y = 4 \qquad (4)$

Multiply (3) by 3, $3x - 21y = 12 \qquad (5)$

Subtract (4) from (5), $y = 8$

Substitute value of y in (3), $x - 56 = 4$

$$\therefore x = 60$$

Therefore the numbers are 60 and 8.

3. A fraction becomes $\frac{4}{5}$ in value by the addition of 2 to its numerator and 3 to its denominator. If 2 is subtracted from its numerator and 1 from its denominator, the value of the fraction is $\frac{3}{4}$. Find the fraction.

Let $\frac{x}{y} =$ the required fraction.

Then
$$\frac{x+2}{y+3} = \frac{4}{5} \qquad (1)$$

and
$$\frac{y-2}{y-1} = \frac{3}{4} \qquad (2)$$

Clear (1) of fractions, $5x + 10 = 4y + 12$

that is, $5x - 4y = 2$ (3)

Clear (2) of fractions, $4x - 8 = 3y - 3$

that is, $4x - 3y = 5$ (4)

Multiply (3) by 3, and (4) by 4.

$$15x - 12y = 6 \quad (5)$$
$$16x - 12y = 20 \quad (6)$$

Subtract (5) from (6), $x = 14$

Substitute value of x in (4).

$$56 - 3y = 5$$
$$3y = 51$$
$$\therefore y = 17$$

Therefore the required fraction is $\frac{14}{17}$.

4. A farmer sold 50 bushels of wheat and 30 of barley for 74 dollars; and at the same prices he sold 30 bushels of wheat and 50 bushels of barley for 70 dollars. What was the price of the wheat and of the barley per bushel?

Let $x =$ the number of dollars the wheat cost a bushel,

and $y =$ the number of dollars the barley cost a bushel.

Then $50x + 30y = 74$

or, $25x + 15y = 37$ (1)

and $30x + 50y = 70$

or, $3x + 5y = 7$ (2)

Multiply (2) by 3, $9x + 15y = 21$ (3)

Subtract (3) from (1), $16x = 16$

$$\therefore x = 1$$

Substitute value of x in (2), $3 + 5y = 7$

$$5y = 4$$
$$\therefore y = \tfrac{4}{5}$$

Wheat, \$1; barley, \$$\tfrac{4}{5}$.

5. If A gave \$10 to B, he would then have three times as much money as B; but if B gave \$5 to A, A would have four times as much as B. How much has each?

Let $x =$ the number of dollars A has,

and $y =$ the number of dollars B has.

Then $x - 10 = 3(y + 10)$ (1)

and $x + 5 = 4(y - 5)$ (2)

(1) is $x - 10 = 3y + 30$

that is $x - 3y = 40$ (3)

(2) is $x + 5 = 4y - 20$

that is $x - 4y = -25$ (4)

Subtract (4) from (3), $y = 65$

Substitute value of y in (3), $x - 195 = 40$

$\therefore x = 235$

A, \$235; B, \$65.

6. A and B have together \$100. If A were to spend one-half of his money, and B one-third of his, they would then have only \$55 between them. How much money has each?

Let $x =$ the number of dollars A has,

and $y =$ the number of dollars B has.

Then $x + y = 100$ (1)

Since A will have $\frac{1}{2}$ of his money left and B $\frac{2}{3}$ of his,

$$\frac{x}{2} + \frac{2y}{3} = 55 \qquad (2)$$

Clear (2) of fractions, $3x + 4y = 330$ (3)

Multiply (1) by 3, $3x + 3y = 300$ (4)

Subtract (4) from (3) $y = 30$

Substitute value of y in (1), $x + 30 = 100$

$\therefore x = 70$

A, \$70; B, \$30.

7. A fruit-dealer sold 6 lemons and 3 oranges for 21 cents, and 3 lemons and 8 oranges for 30 cents. What was the price of each?

Let $x =$ the number of cents a lemon cost,

and $y =$ the number of cents an orange cost.

Then $6x + 3y = 21$ (1)

and $3x + 8y = 30$ (2)

Multiply (2) by 2, $6x + 16y = 60$ (3)

(1) is, $6x + 3y = 21$

Subtract (1) from (3), $13y = 39$

$\therefore y = 3$

Substitute value of y in (1), $6x + 9 = 21$

$6x = 12$

$\therefore x = 2$

Lemon, 2 cents; orange, 3 cents.

8. If A gives me 10 apples, he will have just twice as many as B. If he gives the 10 apples to B instead of to me, A and B will each have the same number. How many apples has each?

Let $x =$ the number of apples A has,
and $y =$ the number of apples B has.

Then $x - 10 = 2y$ (1)
and $x - 10 = y + 10$ (2)
From (1), $x - 2y = 10$ (3)
From (2), $x - y = 20$ (4)
Subtract (3) from (4), $y = 10$
Substitute value of y in (4), $x - 10 = 20$
$\therefore x = 30$

A, 30 apples; B, 10 apples.

Exercise 71. Page 134.

Solve:

1. $5x^2 - 2 = 3x^2 + 6.$

$$5x^2 - 2 = 3x^2 + 6$$
$$5x^2 - 3x^2 = 6 + 2$$
$$2x^2 = 8$$
$$x^2 = 4.$$
$$\therefore x = \pm 2.$$

2. $3x^2 + 1 = 2x^2 + 10.$

$$3x^2 + 1 = 2x^2 + 10$$
$$3x^2 - 2x^2 = 10 - 1$$
$$x^2 = 9.$$
$$\therefore x = \pm 3.$$

3. $4x^2 - 50 = x^2 + 25.$

$$4x^2 - 50 = x^2 + 25$$
$$4x^2 - x^2 = 25 + 50$$
$$3x^2 = 75$$
$$x^2 = 25.$$
$$\therefore x = \pm 5$$

4. $(x - 6)(x + 6) = 28.$

$$(x - 6)(x + 6) = 28$$
$$x^2 - 36 = 28$$
$$x^2 = 28 + 36$$
$$x^2 = 64.$$
$$\therefore x = \pm 8.$$

5. $(x - 5)(x + 5) = 24.$

$$(x - 5)(x + 5) = 24$$
$$x^2 - 25 = 24$$
$$x^2 = 24 + 25$$
$$x^2 = 49.$$
$$\therefore x = \pm 7.$$

6. $3(x^2 - 11) + 2(x^2 - 5) = 82.$

$$3(x^2 - 11) + 2(x^2 - 5) = 82$$
$$3x^2 - 33 + 2x^2 - 10 = 82$$
$$3x^2 + 2x^2 = 82 + 33 + 10$$
$$5x^2 = 125$$
$$x^2 = 25.$$
$$\therefore x = \pm 5.$$

7. $11(x^2+5)+6(3-x^2)=198.$

$$\begin{aligned}11(x^2+5)+6(3-x^2)&=198\\11x^2+55+18-6a^2&=198\\11x^2-6x^2&=198-55-18\\5x^2&=125\\x^2&=25.\\\therefore x&=\pm 5.\end{aligned}$$

8. $5x^2+3-2(17-x^2)=32.$

$$\begin{aligned}5x^2+3-2(17-x^2)&=32\\5x^2+3-34+2x^2&=32\\5x^2+2x^2&=32-3+34\\7x^2&=63\\x^2&=9.\\\therefore x&=\pm 3.\end{aligned}$$

9. $4(x+1)-4(x-1)=x^2-1.$

$$\begin{aligned}4(x+1)-4(x-1)&=x^2-1\\4x+4-4x+4&=x^2-1\\-x^2+4x-4x&=-1-4-4\\-x^2&=-9\\x^2&=9.\\\therefore x&=\pm 3.\end{aligned}$$

10. $86-52x=2(8-x)(2-3x).$

$$\begin{aligned}86-52x&=2(8-x)(2-3x)\\86-52x&=32-52x+6x^2\\-52x+52x-6x^2&=32-86\\6x^2&=54\\x^2&=9.\\\therefore x&=\pm 3.\end{aligned}$$

11. Find two numbers that are to each other as 3 to 4, and the difference of whose squares is 112.

Let $3x =$ the smaller number,

and $4x =$ the larger number.

Then $9x^2 =$ the square of the smaller,

and $16x^2 =$ the square of the larger.

Hence

$$\begin{aligned}16x^2-9x^2&=112\\7x^2&=112\\x^2&=16.\\\therefore x&=\pm 4.\\3x&=12\end{aligned}$$

and $4x=16.$

Therefore the numbers are 12 and 16.

12. A boy bought a number of oranges for 36 cents. The price of an orange was to the number bought as 1 to 4. How many oranges did he buy, and how many cents did each orange cost?

Let $x =$ the number of cents one orange cost,

and $4x =$ the number of oranges.

Then $x+4x =$ the number of cents all cost.

But $36 =$ the number of cents all cost.

$$\begin{aligned}\therefore x\times 4x&=36\\4x^2&=36\\x^2&=9.\\\therefore x&=\pm 3,\end{aligned}$$

and $4x=12.$ 12 oranges at 3 cents each.

13. A certain street contains 144 square rods, and the length is 16 times the width. Find the width.

Let $x =$ the number of rods in width.
Then $16x =$ the number of rods in length,
and $x \times 16x =$ the number of square rods in area.
But $144 =$ the number of square rods in area.

$$\therefore x \times 16x = 144$$
$$16x^2 = 144$$
$$x^2 = 9.$$
$$\therefore x = \pm 3.$$

Therefore the width is 3 rods.

14. Find the number of rods in the length and in the width of a rectangular field containing $3\frac{3}{5}$ acres, if the length is 4 times the width.

Let $x =$ the number of rods in width,
and $4x =$ the number of rods in length.
Then $x \times 4x =$ the number of square rods in area,
$3\frac{3}{5}$ acres $= 576$ square rods.
Hence $576 =$ the number of square rods in area.

$$\therefore x \times 4x = 576$$
$$4x^2 = 576$$
$$x^2 = 144.$$
$$\therefore x = \pm 12$$

and $4x = \pm 48.$

Width, 12 rods; length, 48 rods.

Exercise 72. Page 137.

Solve:

1. $x^2 - 12x + 27 = 0.$

$$x^2 - 12x + 27 = 0$$
$$x^2 - 12x = -27$$
$$x^2 - 12x + 36 = 9$$
$$x - 6 = \pm 3$$
$$x = 6 \pm 3$$
$$\therefore x = 9 \text{ or } 3.$$

2. $x^2 - 6x + 8 = 0.$

$$x^2 - 6x + 8 = 0$$
$$x^2 - 6x = -8$$
$$x^2 - 6x + 9 = 1$$
$$x - 3 = \pm 1$$
$$x = 3 \pm 1$$
$$\therefore x = 4 \text{ or } 2.$$

3. $x^2 - 4 = 4x - 7.$

$$x^2 - 4 = 4x - 7$$
$$x^2 - 4x = -3$$
$$x^2 - 4x + 4 = 1$$
$$x - 2 = \pm 1$$
$$x = 2 \pm 1$$
$$\therefore x = 3 \text{ or } 1.$$

4. $5x^2 - 4x - 1 = 0.$

$$5x^2 - 4x - 1 = 0$$
$$5x^2 - 4x = 1$$
$$x^2 - \tfrac{4}{5}x = \tfrac{1}{5}$$
$$x^2 - \tfrac{4}{5}x + \tfrac{4}{25} = \tfrac{9}{25}$$
$$x - \tfrac{2}{5} = \pm \tfrac{3}{5}$$
$$x = \tfrac{2}{5} \pm \tfrac{3}{5}$$
$$\therefore x = 1 \text{ or } -\tfrac{1}{5}.$$

5. $4x - 3 = 2x - x^2.$

$$4x - 3 = 2x - x^2$$
$$x^2 + 2x = 3$$
$$x^2 + 2x + 1 = 4$$
$$x + 1 = \pm 2$$
$$x = -1 \pm 2$$
$$\therefore x = 1 \text{ or } -3.$$

6. $9x^2 - 24x + 16 = 0.$

$$9x^2 - 24x + 16 = 0$$
$$9x^2 - 24x = -16$$
$$x^2 - \tfrac{8}{3}x = -\tfrac{16}{9}$$
$$x^2 - \tfrac{8}{3}x + \tfrac{16}{9} = 0$$
$$x - \tfrac{4}{3} = 0$$
$$\therefore x = \tfrac{4}{3}.$$

7. $6x^2 - 5x - 1 = 0.$

$$6x^2 - 5x - 1 = 0$$
$$x^2 - \tfrac{5}{6}x = \tfrac{1}{6}$$
$$x^2 - \tfrac{5}{6}x + (\tfrac{5}{12})^2 = \tfrac{49}{144}$$
$$x - \tfrac{5}{12} = \pm \tfrac{7}{12}$$
$$x = \tfrac{5}{12} \pm \tfrac{7}{12}$$
$$\therefore x = 1 \text{ or } -\tfrac{1}{6}.$$

8. $4x + 3 = x^2 + 2x.$

$$4x + 3 = x^2 + 2x$$
$$x^2 - 2x = 3$$
$$x^2 - 2x + 1 = 4$$
$$x - 1 = \pm 2$$
$$x = 1 \pm 2$$
$$\therefore x = 3 \text{ or } -1.$$

9. $16x^2 - 16x + 3 = 0.$

$$16x^2 - 16x + 3 = 0$$
$$16x^2 - 16x = -3$$
$$x^2 - x = -\tfrac{3}{16}$$
$$x^2 - x + \tfrac{1}{4} = \tfrac{1}{16}$$
$$x - \tfrac{1}{2} = \pm \tfrac{1}{4}$$
$$x = \tfrac{1}{2} \pm \tfrac{1}{4}$$
$$\therefore x = \tfrac{3}{4} \text{ or } \tfrac{1}{4}.$$

10. $3x^2 - 10x + 3 = 0.$

$$3x^2 - 10x + 3 = 0$$
$$3x^2 - 10x = -3$$
$$x^2 - \tfrac{10}{3}x = -1$$
$$x^2 - \tfrac{10}{3}x + \tfrac{25}{9} = \tfrac{16}{9}$$
$$x - \tfrac{5}{3} = \pm \tfrac{4}{3}$$
$$x = \tfrac{5}{3} \pm \tfrac{4}{3}$$
$$\therefore x = 3 \text{ or } \tfrac{1}{3}.$$

11. $x^2 - 14x - 51 = 0.$

$$x^2 - 14x - 51 = 0$$
$$x^2 - 14x = 51$$
$$x^2 - 14x + 49 = 100$$
$$x - 7 = \pm 10$$
$$x = 7 \pm 10$$
$$\therefore x = 17 \text{ or } -3.$$

12. $34x - x^2 - 225 = 0.$

$$34x - x^2 - 225 = 0$$
$$x^2 - 34x = -225$$
$$x^2 - 34x + (17)^2 = 64$$
$$x - 17 = \pm 8$$
$$x = 17 \pm 8$$
$$\therefore x = 25 \text{ or } 9.$$

13. $x^2+x-20=0.$

$$\begin{aligned} x^2+x-20&=0\\ x^2+x&=20\\ x^2+x+\tfrac14&=\tfrac{81}{4}\\ x+\tfrac12&=\pm\tfrac92\\ x&=-\tfrac12\pm\tfrac92\\ \therefore x&=4 \text{ or } -5. \end{aligned}$$

14. $x^2-x-12=0.$

$$\begin{aligned} x^2-x-12&=0\\ x^2-x&=12\\ x^2-x+\tfrac14&=\tfrac{49}{4}\\ x-\tfrac12&=\pm\tfrac72\\ x&=\tfrac12\pm\tfrac72\\ \therefore x&=4 \text{ or } -3. \end{aligned}$$

15. $2x^2-12x=-10.$

$$\begin{aligned} 2x^2-12x&=-10\\ x^2-6x&=-5\\ x^2-6x+9&=4\\ x-3&=\pm2\\ x&=3\pm2\\ \therefore x&=5 \text{ or } 1. \end{aligned}$$

16. $3x^2+12x-36=0.$

$$\begin{aligned} 3x^2+12x-36&=0\\ x^2+4x&=12\\ x^2+4x+4&=16\\ x+2&=\pm4\\ x&=-2\pm4\\ \therefore x&=2 \text{ or } -6. \end{aligned}$$

17. $(2x-1)^2+9=6(2x-1).$

$$\begin{aligned} (2x-1)^2+9&=6(2x-1)\\ 4x^2-4x+10&=12x-6\\ 4x^2-16x&=-16\\ x^2-4x&=-4\\ x^2-4x+4&=0\\ x-2&=0\\ \therefore x&=2. \end{aligned}$$

18. $6(9x^2-x)=55(x^2-1).$

$$\begin{aligned} 6(9x^2-x)&=55(x^2-1)\\ 54x^2-6x&=55x^2-55\\ x^2+6x&=55\\ x^2+6x+9&=64\\ x+3&=\pm8\\ x&=-3\pm8\\ \therefore x&=5 \text{ or } -11. \end{aligned}$$

19. $32-3x^2-10x=0.$

$$\begin{aligned} 32-3x^2-10x&=0\\ 3x^2+10x&=32\\ x^2+\frac{10x}{3}&=\frac{32}{3}\\ x^2+\frac{10x}{3}+\frac{25}{9}&=\frac{121}{9}\\ x+\tfrac53&=\pm\tfrac{11}{3}\\ x&=-\tfrac53\pm\tfrac{11}{3}\\ \therefore x&=2 \text{ or } -5\tfrac13. \end{aligned}$$

20. $9x^2-6x-143=0.$

$$\begin{aligned} 9x^2-6x-143&=0\\ 9x^2-6x&=143\\ x^2-\tfrac23x&=\tfrac{143}{9}\\ x^2-\tfrac23x+\tfrac19&=\tfrac{144}{9}\\ x-\tfrac13&=\pm\tfrac{12}{3}\\ x&=\tfrac13\pm\tfrac{12}{3}\\ \therefore x&=4\tfrac13 \text{ or } -3\tfrac23. \end{aligned}$$

21. $\dfrac{x}{x-1}-\dfrac{x-1}{x}=\dfrac32.$

$$\begin{aligned} \frac{x}{x-1}-\frac{x-1}{x}&=\frac32\\ 2x\times x-2(x-1)^2&=3x^2-3x\\ 2x^2-2x^2+4x-2&=3x^2-3x\\ -3x^2+7x&=2\\ x^2-\frac{7x}{3}&=-\frac23\\ x^2-\frac{7x}{3}+\frac{49}{36}&=\frac{25}{36} \end{aligned}$$

$$\begin{aligned} x - \tfrac{7}{6} &= \pm \tfrac{5}{6} \\ x &= \tfrac{7}{6} \pm \tfrac{5}{6} \\ \therefore x &= 2 \text{ or } \tfrac{1}{3}. \end{aligned}$$

22. $\dfrac{1}{x-2} + \dfrac{2}{x+2} = \dfrac{5}{6}.$

$$\begin{aligned} \frac{1}{x-2} + \frac{2}{x+2} &= \frac{5}{6} \\ \frac{3x-2}{x^2-4} &= \frac{5}{6} \\ 5x^2 - 20 &= 18x - 12 \\ 5x^2 - 18x &= 8 \\ x^2 - \frac{18x}{5} &= \frac{8}{5} \\ x^2 - \frac{18x}{5} + \frac{81}{25} &= \frac{121}{25} \\ x - \tfrac{9}{5} &= \pm \tfrac{11}{5} \\ x &= \tfrac{9}{5} \pm \tfrac{11}{5} \\ \therefore x &= 4 \text{ or } -\tfrac{2}{5}. \end{aligned}$$

23. $\dfrac{5x+7}{x-1} = 3x + 11.$

$$\begin{aligned} \frac{5x+7}{x-1} &= 3x + 11 \\ 5x + 7 &= 3x^2 + 8x - 11 \\ 3x^2 + 3x &= 18 \\ x^2 + x &= 6 \\ x^2 + x + \tfrac{1}{4} &= \tfrac{25}{4} \\ x + \tfrac{1}{2} &= \pm \tfrac{5}{2} \\ x &= -\tfrac{1}{2} \pm \tfrac{5}{2} \\ \therefore x &= 2 \text{ or } -3. \end{aligned}$$

24. $\dfrac{7}{x+4} - \dfrac{1}{4-x} = \dfrac{2}{3}.$

$$\begin{aligned} \frac{7}{x+4} - \frac{1}{4-x} &= \frac{2}{3} \\ \frac{24-8x}{16-x^2} &= \frac{2}{3} \\ 72 - 24x &= 32 - 2x^2 \\ 2x^2 - 24x &= -40 \\ x^2 - 12x &= -20 \\ x^2 - 12x + 36 &= 16 \\ x - 6 &= \pm 4 \\ x &= 6 \pm 4 \\ \therefore x &= 10 \text{ or } 2. \end{aligned}$$

25. $\dfrac{2}{x+3} + \dfrac{x+3}{2} = \dfrac{10}{3}.$

$$\begin{aligned} \frac{2}{x+3} + \frac{x+3}{2} &= \frac{10}{3} \\ 12 + 3x^2 + 18x + 27 &= 20x + 60 \\ 3x^2 - 2x &= 21 \\ x^2 - \tfrac{2}{3}x &= 7 \\ x^2 - \tfrac{2}{3}x + \tfrac{1}{9} &= \tfrac{64}{9} \\ x - \tfrac{1}{3} &= \pm \tfrac{8}{3} \\ x &= \tfrac{1}{3} \pm \tfrac{8}{3} \\ \therefore x &= 3 \text{ or } -2\tfrac{1}{3}. \end{aligned}$$

26. $\dfrac{2x}{x+2} + \dfrac{x+2}{2x} = 2.$

$$\begin{aligned} \frac{2x}{x+2} + \frac{x+2}{2x} &= 2 \\ 4x^2 + x^2 + 4x + 4 &= 4x^2 + 8x \\ x^2 - 4x &= -4 \\ x^2 - 4x + 4 &= 0 \\ x - 2 &= 0 \\ \therefore x &= 2. \end{aligned}$$

27 $\dfrac{3(x-1)}{x+1} - \dfrac{2(x+1)}{x-1} = 5.$

$$\begin{aligned} \frac{3(x-1)}{x+1} - \frac{2(x+1)}{x-1} &= 5 \\ 3x^2 - 6x + 3 - 2x^2 - 4x - 2 &= 5x^2 - 5 \\ -4x^2 - 10x &= -6 \\ x^2 + \tfrac{5}{2}x &= \tfrac{3}{2} \\ x^2 + \tfrac{5}{2}x + \tfrac{25}{16} &= \tfrac{49}{16} \\ x + \tfrac{5}{4} &= \pm \tfrac{7}{4} \\ x &= -\tfrac{5}{4} \pm \tfrac{7}{4} \\ \therefore x &= \tfrac{1}{2} \text{ or } -3. \end{aligned}$$

28. $\dfrac{2x+5}{2x-5}=\dfrac{7x-5}{2x}.$

$$\frac{2x+5}{2x-5}=\frac{7x-5}{2x}$$
$$14x^2-45x+25=4x^2+10x$$
$$10x^2-55x=-25$$
$$2x^2-11x=-5$$
$$x^2-\frac{11x}{2}=-\frac{5}{2}$$
$$x^2-\frac{11x}{2}+\frac{121}{16}=\frac{81}{16}$$
$$x-\tfrac{11}{4}=\pm\tfrac{9}{4}$$
$$x=\tfrac{11}{4}\pm\tfrac{9}{4}$$
$$\therefore x=5 \text{ or } \tfrac{1}{2}.$$

29. $\dfrac{3x-1}{4x+7}=\dfrac{x+1}{x+7}.$

$$\frac{3x-1}{4x+7}=\frac{x+1}{x+7}$$
$$4x^2+11x+7=3x^2+20x-7$$
$$x^2-9x=-14$$
$$x^2-9x+\tfrac{81}{4}=\tfrac{25}{4}$$
$$x-\tfrac{9}{2}=\pm\tfrac{5}{2}$$
$$x=\tfrac{9}{2}\pm\tfrac{5}{2}$$
$$\therefore x=7 \text{ or } 2.$$

30. $\dfrac{2x-1}{x+3}=\dfrac{x+3}{2x-1}.$

$$\frac{2x-1}{x+3}=\frac{x+3}{2x-1}$$
$$4x^2-4x+1=x^2+6x+9$$
$$3x^2-10x=8$$
$$x^2-\tfrac{10}{3}x=\tfrac{8}{3}$$
$$x^2-\frac{10x}{3}+\frac{25}{9}=\frac{49}{9}$$
$$x-\tfrac{5}{3}=\pm\tfrac{7}{3}$$
$$x=\tfrac{5}{3}\pm\tfrac{7}{3}$$
$$\therefore x=4 \text{ or } -\tfrac{2}{3}$$

31. $\dfrac{x+4}{x-4}-\dfrac{x+2}{x-3}=1.$

$$\frac{x+4}{x-4}-\frac{x+2}{x-3}=1$$
$$x^2+x-12-x^2+2x+8=x^2-7x+12$$
$$x^2-10x=-16$$
$$x^2-10x+25=9$$
$$x-5=\pm 3$$
$$x=5\pm 3$$
$$\therefore x=8 \text{ or } 2.$$

32. $\dfrac{4}{x-1}-\dfrac{5}{x+2}=\dfrac{1}{2}.$

$$\frac{4}{x-1}-\frac{5}{x+2}=\frac{1}{2}$$
$$8x+16-10x+10=x^2+x-2$$
$$x^2+3x=28$$
$$x^2+3x+\tfrac{9}{4}=\tfrac{121}{4}$$
$$x+\tfrac{3}{2}=\pm\tfrac{11}{2}$$
$$x=-\tfrac{3}{2}\pm\tfrac{11}{2}$$
$$\therefore x=4 \text{ or } -7.$$

33. $\dfrac{2}{x-1}=\dfrac{3}{x-2}+\dfrac{2}{x-4}.$

$$\frac{2}{x-1}=\frac{3}{x-2}+\frac{2}{x-4}$$
$$2x^2-12x+16=3x^2-15x+12+2x^2-6x+4$$
$$-3x^2+9x=0$$
$$x^2-3x=0$$
$$x^2-3x+\tfrac{9}{4}=\tfrac{9}{4}$$
$$x-\tfrac{3}{2}=\pm\tfrac{3}{2}$$
$$x=\tfrac{3}{2}\pm\tfrac{3}{2}$$
$$\therefore x=3 \text{ or } 0.$$

34. $\dfrac{5}{x-2}-\dfrac{3}{x-1}=\dfrac{1}{2}.$

$$\frac{5}{x-2}-\frac{3}{x-1}=\frac{1}{2}$$
$$10x-10-6x+12=x^2-3x+2$$
$$x^2-7x=0$$
$$x^2-7x+\tfrac{49}{4}=\tfrac{49}{4}$$
$$x-\tfrac{7}{2}=\pm\tfrac{7}{2}$$
$$x=\tfrac{7}{2}\pm\tfrac{7}{2}$$
$$\therefore x=7 \text{ or } 0.$$

35. $\dfrac{x}{7-x}+\dfrac{7-x}{x}=\dfrac{29}{10}.$

$$\frac{x}{7-x}+\frac{7-x}{x}=\frac{29}{10}$$
$$10x^2+490-140x+10x^2=203x-29x^2$$
$$49x^2-343x=-490$$

Divide by 49, $x^2-7x=-10$

$$x^2-7x+\tfrac{49}{4}=\tfrac{9}{4}$$
$$x-\tfrac{7}{2}=\pm\tfrac{3}{2}$$
$$x=\tfrac{7}{2}\pm\tfrac{3}{2}$$
$$\therefore x=5 \text{ or } 2.$$

36. $\dfrac{2x-1}{x-1}+\dfrac{1}{6}=\dfrac{2x-3}{x-2}.$

$$\frac{2x-1}{x-1}+\frac{1}{6}=\frac{2x-3}{x-2}$$
$$12x^2-30x+12+x^2-3x+2=12x^2-30x+18$$
$$x^2-3x=4$$
$$x^2-3x+\tfrac{9}{4}=\tfrac{25}{4}$$
$$x-\tfrac{3}{2}=\pm\tfrac{5}{2}$$
$$x=\tfrac{3}{2}\pm\tfrac{5}{2}$$
$$\therefore x=4 \text{ or } -1.$$

Exercise 73. Page 140.

1. Find two numbers whose sum is 11, and whose product is 30.

Let $x=$ one number.

Then $11-x=$ the other number,

and $\quad x(11-x) =$ the product of the numbers,

but $\quad 30 =$ the product of the numbers.

Therefore

$$x(11-x)=30$$
$$11x-x^2=30$$
$$x^2-11x=-30$$
$$x^2-11x+\tfrac{121}{4}=\tfrac{1}{4}$$
$$x-\tfrac{11}{2}=\pm\tfrac{1}{2}$$
$$x=\tfrac{11}{2}\pm\tfrac{1}{2}$$
$$\therefore x=6 \text{ or } 5.$$

Therefore the two numbers are 6 and 5.

2. Find two numbers whose difference is 10, and the sum of whose squares is 250.

Let $\quad x =$ one number.

Then $\quad x+10 =$ the other number,

and $\quad x^2+(x+10)^2 =$ the sum of their squares,

but $\quad 250 =$ the sum of their squares.

Therefore

$$x^2+(x+10)^2=250$$
$$2x^2+20x+100=250$$
$$2x^2+20x=150$$
$$x^2+10x=75$$
$$x^2+10x+25=100$$
$$x+5=\pm 10$$
$$x=-5\pm 10$$
$$\therefore x=5 \text{ or } -15.$$

Therefore the numbers are 5 and 15.

3. A man is five times as old as his son, and the square of the son's age diminished by the father's age is 24. Find their ages.

Let $\quad x =$ the son's age,

and $\quad 5x =$ the father's age.

Then

$$x^2-5x=24$$
$$x^2-5x+\tfrac{25}{4}=\tfrac{121}{4}$$
$$x-\tfrac{5}{2}=\pm\tfrac{11}{2}$$
$$x=\tfrac{5}{2}\pm\tfrac{11}{2}$$
$$\therefore x=8 \text{ or } -3.$$

Therefore the son is 8 years old, and the father is 40.

4. A number increased by its square is equal to nine times the next higher number. Find the number.

Let $x =$ the number.

Then $x + 1 =$ the next higher number,

$x + x^2 =$ the number plus its square,

and $9(x + 1) =$ nine times the next higher number.

As these two expressions are equal we have,

$$x + x^2 = 9(x + 1)$$
$$x + x^2 = 9x + 9$$
$$x^2 - 8x = 9$$
$$x^2 - 8x + 16 = 25$$
$$x - 4 = \pm 5$$
$$x = 4 \pm 5$$
$$\therefore x = 9 \text{ or } -1.$$

Therefore the number is 9.

5. The square of the sum of any two consecutive numbers lacks 1 of being twice the sum of the squares of the numbers. Show that this statement is true.

Let $x =$ the smaller number.

Then $x + 1 =$ the larger number,

and $2x + 1 =$ the sum of the numbers.

Therefore $(2x + 1)^2 + 1 = 2\{x^2 + (x + 1)^2\}$.

Therefore $4x^2 + 4x + 1 + 1 = 2\{x^2 + x^2 + 2x + 1\}$

or, $4x^2 + 4x + 2 = 4x^2 + 4x + 2$.

This equation is identical, and hence true for any value of x.

6. The length of a rectangular court exceeds its breadth by 2 rods. If the length and breadth were each increased by 3 rods, the area of the court would be 80 square rods. Find the dimensions of the court.

Let $x =$ the breadth in rods.

Then $x + 2 =$ the length in rods.

If each dimension were increased by 3 rods, the breadth would be $x + 3$, and the length $x + 2 + 3$, or $x + 5$ rods. The area would be $(x + 5)(x + 3)$.

Therefore $(x + 5)(x + 3) = 80$

$$x^2 + 8x + 15 = 80$$
$$x^2 + 8x = 65$$

$$x^2 + 8x + 16 = 81$$
$$x + 4 = \pm 9.$$
$$x = -4 \pm 9$$
$$\therefore x = 5 \text{ or } -13.$$

Therefore the dimensions are 5 by 7 rods.

7. The area of a certain square will be doubled, if its dimensions are increased by 6 feet and 4 feet respectively. Find its dimensions.

Let $x =$ the side of the square.

Then $x^2 =$ the area of the square,

$(x + 6)(x + 4) =$ twice the area of the square.

Therefore
$$(x + 6)(x + 4) = 2x^2$$
$$x^2 + 10x + 24 = 2x^2$$
$$x^2 - 10x = 24$$
$$x^2 - 10x + 25 = 49$$
$$x - 5 = \pm 7$$
$$x = 5 \pm 7$$
$$\therefore x = 12 \text{ or } -2.$$

Therefore a side of the square is 12 feet.

8. The perimeter of a rectangular floor is 76 feet and the area of the floor is 360 square feet. Find the dimensions of the floor.

Since the two sides and two ends are 76 feet, one side and one end is 38 feet.

Let $x =$ the length of the floor.

Then $38 - x =$ the breadth of the floor,

and $x(38 - x) =$ the area of the floor.

Therefore
$$x(38 - x) = 360$$
$$38x - x^2 = 360$$
$$x^2 - 38x = -360$$
$$x^2 - 38x + (19)^2 = 1$$
$$x - 19 = \pm 1$$
$$x = 19 \pm 1$$
$$\therefore x = 20 \text{ or } 18.$$

Therefore the dimensions are 20 feet and 18 feet.

9. The length of a rectangular court exceeds its breadth by 2 rods, and its area is 120 square rods. Find the dimensions of the court.

Let $x =$ the breadth of the court in rods.

Then $x + 2 =$ the length of the court in rods,

and $x(x + 2) =$ the area of the court in square rods.

Therefore
$$x^2 + 2x = 120$$
$$x^2 + 2x + 1 = 121$$
$$x + 1 = \pm 11$$
$$x = -1 \pm 11$$
$$\therefore x = 10 \text{ or } -12.$$

Therefore the dimensions are 10 by 12 rods.

10. The combined ages of a father and son amount to 64 years. Twice the father's age exceeds the square of the son's age by 8 years. Find their respective ages.

Let $x =$ the son's age.

Then $64 - x =$ the father's age.

$$2(64 - x) - x^2 = 8$$
$$128 - 2x - x^2 = 8$$
$$x^2 + 2x = 120$$
$$x^2 + 2x + 1 = 121$$
$$x + 1 = \pm 11$$
$$x = -1 \pm 11$$
$$\therefore x = 10 \text{ or } -12.$$

Therefore the son is 10 and the father 54 years old.

Exercise 74. Page 141.

1. A boat sails 30 miles at a uniform rate. If the rate had been 1 mile an hour less, the time of the sailing would have been 1 hour more. Find the rate of the sailing.

Let $x =$ the rate of sailing per hour.

Then $\frac{30}{x} =$ the number of hours.

On the other supposition the rate would have been $x - 1$ miles an hour and the time $\frac{30}{x-1}$.

Hence $\frac{30}{x-1} - \frac{30}{x} =$ the difference in hours for sailing.

But $1 =$ the difference in hours for sailing.

$$\therefore \frac{30}{x-1} - \frac{30}{x} = 1$$
$$30x - 30x + 30 = x^2 - x$$
$$x^2 - x = 30$$

$$x^2 - x + \tfrac{1}{4} = \tfrac{121}{4}$$
$$x - \tfrac{1}{2} = \pm \tfrac{11}{2}$$
$$x = \tfrac{1}{2} \pm \tfrac{11}{2}$$
$$\therefore x = 6 \text{ or } -5.$$

Therefore the boat sails 6 miles an hour.

2. A laborer built 35 rods of stone wall. If he had built 2 rods less each day, it would have taken him 2 days longer. How many rods did he build a day on the average?

Let $x =$ the number of rods he built a day.

Then $\frac{35}{x} =$ the number of days he worked.

On the other supposition he would have built $x - 2$ rods a day, and it would have taken him $\frac{35}{x-2}$ days.

Hence $\frac{35}{x-2} - \frac{35}{x} =$ the difference in the number of days.

But $2 =$ the difference in the number of days.

$$\therefore \frac{35}{x-2} - \frac{35}{x} = 2$$
$$35x - 35x + 70 = 2x^2 - 4x$$
$$2x^2 - 4x = 70$$
$$x^2 - 2x = 35$$
$$x^2 - 2x + 1 = 36$$
$$x - 1 = \pm 6$$
$$x = 1 \pm 6$$
$$\therefore x = 7 \text{ or } -5.$$

Therefore he built 7 rods a day.

3. A man bought flour for \$30. Had he bought 1 barrel more for the same sum, the flour would have cost him \$1 less per barrel. How many barrels did he buy?

Let $x =$ the number of barrels he bought.

Then $\frac{30}{x} =$ the number of dollars he paid a barrel.

On the other supposition he would have bought $x + 1$ barrels, and paid $\frac{30}{x+1}$ dollars per barrel.

Therefore $\frac{30}{x} - \frac{30}{x+1} =$ the difference in price in dollars.

But $1 =$ the difference in price in dollars.

$$\therefore \frac{30}{x} - \frac{30}{x+1} = 1$$
$$30x + 30 - 30x = x^2 + x$$
$$x^2 + x = 30$$
$$x^2 + x + \tfrac{1}{4} = \tfrac{121}{4}$$
$$x + \tfrac{1}{2} = \pm \tfrac{11}{2}$$
$$x = -\tfrac{1}{2} \pm \tfrac{11}{2}$$
$$\therefore x = 5 \text{ or } -6.$$

Therefore he bought 5 barrels.

4. A man bought some knives for $6. Had he bought 2 less for the same money, he would have paid 25 cents more for each knife. How many knives did he buy?

Let $x =$ the number of knives he bought.

Then $\frac{6}{x} =$ the number of dollars he paid for each one.

On the other supposition he would have bought $x - 2$, and paid $\frac{6}{x-2}$ dollars for each. The difference in price would have been 25 cents; that is, $\frac{1}{4}$ of a dollar.

Therefore
$$\frac{6}{x-2} - \frac{6}{x} = \frac{1}{4}$$
$$24x - 24x + 48 = x^2 - 2x$$
$$x^2 - 2x = 48$$
$$x^2 - 2x + 1 = 49$$
$$x - 1 = \pm 7$$
$$x = 1 \pm 7$$
$$\therefore x = 8 \text{ or } -6.$$

Therefore he bought 8 knives.

5. What number exceeds its square root by 30?

Let $x^2 =$ the number.

Then $x =$ the square root of the number,

and
$$x^2 - x = 30.$$
$$x^2 - x + \tfrac{1}{4} = \tfrac{121}{4}$$
$$x - \tfrac{1}{2} = \pm \tfrac{11}{2}$$
$$x = \tfrac{1}{2} \pm \tfrac{11}{2}$$
$$x = 6 \text{ or } -5.$$

Therefore $x = 6,\ x^2 = 36.$

Therefore the number is 36.

Exercise 75. Page 144.

1. Find the 25th term in the series 3, 6, 9, ...

Here $a = 3,\ d = 3, \text{ and } n = 25.$

$$\therefore l = 3 + (25 - 1)(3)$$
$$= 75.$$

2. Find the 13th term in the series 50, 49, 48, ...

Here $a = 50,\ d = -1, \text{ and } n = 13.$

$$\therefore l = 50 + (13 - 1)(-1)$$
$$= 38.$$

3. Find the 15th term in the series $\frac{1}{7}$, $\frac{3}{7}$, $\frac{5}{7}$, ...

Here $a = \frac{1}{7},\ d = \frac{2}{7}, \text{ and } n = 15.$

$$\therefore l = \tfrac{1}{7} + (15 - 1)(\tfrac{2}{7})$$
$$= 4\tfrac{1}{7}.$$

4. Find the 19th term in the series $\frac{1}{4}$, $-\frac{1}{4}$, $-\frac{3}{4}$, ...

Here $a = \frac{1}{4},\ d = -\frac{1}{2},\ n = 19.$

$$\therefore l = \tfrac{1}{4} + (19 - 1)(-\tfrac{1}{2})$$
$$= -8\tfrac{3}{4}.$$

5. Find the 10th term in an arithmetical progression whose 1st term is 5 and 3d term 9.

Here $a = 5$

$$a + d = \frac{5 + 9}{2}$$
$$= 7.$$
$$\therefore d = 2$$
$$n = 10.$$
$$\therefore l = 5 + (10 - 1)(2)$$
$$= 23.$$

6. Find the 11th term in an arithmetical progression whose 1st term is 10 and whose 6th term is 5.

Here $a = 10$, the 6th term $= a + 5d.$

And since the 6th term is 5,

$$a + 5d = 5.$$

Hence $10 + 5d = 5,$

and $5d = -5.$

Therefore $d = -1.$

Also $n = 11.$

$$\therefore l = 10 + (11 - 1)(-1)$$
$$= 0.$$

7. If the 3d term of an arithmetical progression is 20 and the 13th term is 100, what is the 20th term?

$$a + 2d = \text{the 3d term,}$$

and $$a + 12d = \text{the 13th term.}$$

$$\therefore a + 12d - (a + 2d) = 100 - 20$$
$$10d = 80$$
$$d = 8.$$

Since $$a + 2d = 20$$
$$a = 4.$$
$$\therefore l = 4 + (20 - 1)(8)$$
$$= 156.$$

8. Which term of the series 5, 7, 9, 11, ..., is 43?

Here, $a = 5$, $d = 2$, and $l = 43$.

Substitute these values in the formula

$$l = a + (n - 1)d$$

and we have, $$43 = 5 + (n - 1)2$$
$$43 = 5 + 2n - 2.$$
$$\therefore 40 = 2n$$
$$n = 20.$$

9. Which term of the series $\frac{4}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, ..., is 18?

Here $a = \frac{4}{3}$, $d = \frac{1}{6}$, and $l = 18$.

Therefore $$18 = \tfrac{4}{3} + (n - 1)\left(\tfrac{1}{6}\right)$$
$$18 = \frac{4}{3} + \frac{n}{6} - \frac{1}{6}$$
$$108 = 8 + n - 1.$$
$$\therefore n = 101.$$

10. What is the arithmetical mean of 20 and 32?

The arithmetical mean between 20 and 32 is $\dfrac{20 + 32}{2} = 26$.

11. What is the arithmetical mean of $a + b$ and $a - b$?

The arithmetical mean between $a + b$ and $a - b$ is $\dfrac{a + b + a - b}{2} = a$.

12. Insert 8 arithmetical means between 20 and 29.

In the formula $\dfrac{l - a}{m + 1} = d$, $l = 29$, $a = 20$, and $m = 8$.

$$\therefore \frac{l - a}{m + 1} = \frac{29 - 20}{9} = 1 = d.$$

Hence, the series is 20, 21, 22, 23, 24, 25, 26, 27, 28, 29.

Exercise 76. Page 146.

1. Find the sum of 3, 5, 7, ..., to 20 terms.

Formula (3) is $s = \frac{n}{2}\{2a + (n-1)d\}$.

Here $a = 3$, $d = 2$, and $n = 20$.

$$\begin{aligned} \therefore s &= \tfrac{20}{2}\{6 + (20-1)(2)\} \\ &= 10 \times 44 \\ &= 440. \end{aligned}$$

2. Find the sum of 14, $14\frac{1}{2}$, 15, ..., to 12 terms.

Here $a = 14$, $d = \frac{1}{2}$, and $n = 12$.

$$\begin{aligned} \therefore s &= \tfrac{12}{2}\{28 + (12-1)(\tfrac{1}{2})\} \\ &= 6 \times 33\tfrac{1}{2} \\ &= 201. \end{aligned}$$

3. Find the sum of $\frac{7}{6}$, 1, $\frac{5}{6}$, ..., to 10 terms.

Here $a = \frac{7}{6}$, $d = -\frac{1}{6}$, and $n = 10$.

$$\begin{aligned} \therefore s &= \tfrac{10}{2}\{\tfrac{7}{3} + (10-1)(-\tfrac{1}{6})\} \\ &= 5 \times \tfrac{5}{6} \\ &= 4\tfrac{1}{6}. \end{aligned}$$

4. Find the sum of -7, -5, -3, ..., to 16 terms.

Here $a = -7, d = 2$, and $n = 16$.

$$\begin{aligned} \therefore s &= \tfrac{16}{2}\{-14 + (16-1)2\} \\ &= 8 \times 16 \\ &= 128. \end{aligned}$$

5. Find the sum of 12, 9, 6, ..., to 21 terms.

Here $a = 12$, $d = -3$, $n = 21$.

$$\begin{aligned} \therefore s &= \tfrac{21}{2}\{24 + (21-1)(-3)\} \\ &= -378. \end{aligned}$$

6. Find the sum of $-10\frac{1}{2}$, -9, $-7\frac{1}{2}$, ..., to 25 terms.

Here $a = -10\frac{1}{2}$, $d = 1\frac{1}{2}$, and $n = 25$.

$$\begin{aligned} \therefore s &= \tfrac{25}{2}\{-21 + (25-1)(1\tfrac{1}{2})\} \\ &= 12\tfrac{1}{2} \times 15 \\ &= 187\tfrac{1}{2}. \end{aligned}$$

7. The sum of three numbers in arithmetical progression is 9, and the sum of their squares is 35. Find the numbers.

Let $x - y$, x, $x + y$, stand for the numbers.

Then $\qquad x - y + x + x + y = 9, \qquad (1)$

and $\qquad (x - y)^2 + x^2 + (x + y)^2 = 35. \qquad (2)$

From (1), $\qquad 3x = 9.$

$$\therefore x = 3.$$

Put 3 for x in (2), and we have

$$9 - 6y + y^2 + 9 + 9 + 6y + y^2 = 35$$
$$2y^2 = 8$$
$$y^2 = 4$$
$$y = \pm 2.$$

Therefore the numbers are 1, 3, and 5.

8. A common clock strikes the hours from 1 to 12. How many times does it strike every 24 hours?

Here $\qquad a = 1,\ d = 1,\ n = 12.$

$$\therefore s = \tfrac{12}{2}\{2 + (12 - 1)(1)\}$$
$$= 6 \times 13$$
$$= 78 \text{ for 12 hours,}$$

and $\qquad 2 \times 78 = 156$ for 24 hours.

9. The Greenwich clock strikes the hours from 1 to 24. How many times does it strike in 24 hours?

$$a = 1,\ d = 1,\ n = 24.$$
$$\therefore s = \tfrac{24}{2}\{2 + (24 - 1)(1)\}$$
$$= 12 \times 25$$
$$= 300.$$

10. In a potato race each man picked up 50 potatoes placed in line a yard apart, and the first potato one yard from the basket, picking up one potato at a time and bringing it to the basket. How many yards did each man run, the start being made from the basket?

Here $\qquad a = 2,\ d = 2, \text{ and } n = 50.$

Therefore $\qquad s = \tfrac{50}{2}\{4 + (50 - 1)2\}$

$$= 25 \times 102$$
$$= 2550 \text{ yards.}$$

11. A heavy body falling from a height falls 16.1 feet the first second, and in each succeeding second 32.2 feet more than in the second next preceding. How far will a body fall in 19 seconds?

Here $a = 16.1,\ d = 32.2$, and $n = 19$.

$$\therefore s = \tfrac{19}{2}\{32.2 + (19 - 1)\,32.2\}$$
$$= 5812.1 \text{ feet.}$$

12. A stone dropped from a bridge reached the water in just 3 seconds. Find the height of the bridge. (See Ex. 11.)

Here $a = 16.1,\ d = 32.2$, and $n = 3$.

$$\therefore s = \tfrac{3}{2}\{32.2 + (3 - 1)\,32.2\}$$
$$= 144.9 \text{ feet.}$$

13. The arithmetical mean between two numbers is 13, and the mean between the double of the first and the triple of the second is $33\tfrac{1}{2}$. Find the numbers.

Let x and y stand for the numbers.

Then $$\frac{x+y}{2} = 13,$$

and $$\frac{2x+3y}{2} = 33\tfrac{1}{2}.$$

$$\therefore x + y = 26,$$

or $$2x + 2y = 52,$$

and $$2x + 3y = 67.$$

$$\therefore y = 15,$$

and $$x = 11.$$

14. Find three numbers of an arithmetical series whose sum shall be 27, and the sum of the first and second shall be $\tfrac{4}{5}$ of the sum of the second and third.

Let $x - y$, x, and $x + y$ stand for the numbers.

Then $x - y + x + x + y = 27$.

$$\therefore 3x = 27,$$

and $$x = 9.$$

$$x - y + x = \tfrac{4}{5}(2x + y)$$
$$5(2x - y) = 4(2x + y)$$
$$10x - 5y = 8x + 4y$$
$$2x = 9y.$$
$$\therefore 9y = 2x$$
$$= 18$$

$$\therefore y = 2$$

Hence $$x - y = 7$$
$$x = 9$$
$$x + y = 11.$$

Therefore the numbers are 7, 9, and 11.

15. A travels uniformly 20 miles a day; B travels 8 miles the first day, 12 the second, and so on, in arithmetical progression. If they start Monday morning from the same place and travel in the same direction, how far apart will they be Saturday night?

A travels 6×20 miles $= 120$ miles.

Since $$a = 8,\ d = 4,\ n = 6$$
$$s = \tfrac{6}{2}\{16 + (6-1)4\}$$
$$= 3 \times 36$$
$$= 108 \text{ miles},$$
$$120 - 108 = 12.$$

Therefore they will be 12 miles apart.

16. The sum of three terms of an arithmetical progression is 36, and the square of the means exceeds the product of the other two terms by 49. Find the numbers.

Let $x - y$, x, and $x + y$ stand for the numbers.

Then $x - y + x + x + y = 36.$
$$\therefore x = 12$$

Hence $$144 - 49 = (12 - y)(12 + y)$$
$$y^2 = 49,$$

and $$y = \pm 7.$$

Therefore the numbers are 5, 12, and 19.

Exercise 77. Page 151.

1. Find the 5th term of 3, 9, 27, ...

Here $$a = 3,\ r = 3,\ n = 5.$$
$$\therefore l = 3 \times 3^4 = 243.$$

2. Find the 7th term of 3, 6, 12, ...

Here $$a = 3,\ r = 2,\ n = 7.$$
$$\therefore l = 3 + 2^6 = 192.$$

3. Find the 8th term of 6, 3, $\frac{3}{2}$, ...

Here $$a = 6,\ r = \tfrac{1}{2},\ n = 8.$$
$$\therefore l = 6\left(\tfrac{1}{2}\right)^7 = \tfrac{3}{64}.$$

4. Find the 9th term of 1, − 2, 4, ...

Here $$a = 1,\ r = -2,\ n = 9.$$
$$\therefore l = 1\,(-2)^8 = 256.$$

5. Find the geometrical mean between 2 and 8.

The geometrical mean is $\sqrt{2 \times 8} = \sqrt{16} = \pm 4$.

6. Find the common ratio if the first and 3d terms are 2 and 32.

Here $$a = 2, \text{ and } ar^2 = 32.$$
$$\therefore r^2 = 16,$$
and $$r = \pm 4.$$

Find the sum of the series:

7. 3, 9, 27, ... to 6 terms.

Here $$a = 3,\ r = 3,\ n = 6.$$
$$\therefore s = \frac{a\,(r^n - 1)}{r - 1} = \frac{3\,(3^6 - 1)}{2} = 1092.$$

8. 3, 6, 12, ... to 8 terms.

Here $$a = 3,\ r = 2,\ n = 8.$$
$$\therefore s = \frac{3\,(2^8 - 1)}{1} = 765.$$

9. 6, 3, $\frac{3}{2}$, ... to 7 terms.

Here $$a = 6,\ r = \tfrac{1}{2},\ n = 7.$$
$$\therefore s = \frac{6\{1 - (\frac{1}{2})^7\}}{\frac{1}{2}} = 11\tfrac{29}{32}.$$

10. 8, 4, 2, ... to 8 terms.

Here $$a = 8,\ r = \tfrac{1}{2},\ n = 8.$$
$$\therefore s = \frac{8\,\{1 - (\frac{1}{2})^8\}}{\frac{1}{2}} = 15\tfrac{15}{16}.$$

11. 64, 32, 16, ... to 9 terms.

Here $$a = 64,\ r = \tfrac{1}{2},\ n = 9.$$
$$\therefore s = \frac{64\,\{1 - (\frac{1}{2})^9\}}{\frac{1}{2}} = 127\tfrac{3}{4}.$$

12. 64, − 32, 16, ... to 5 terms.

Here $$a = 64,\ r = -2,\ n = 5.$$
$$\therefore s = \frac{64\,\{1 - (-\frac{1}{2})^5\}}{\frac{3}{2}} = 44.$$

13. $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \ldots\ldots$ to 4 terms.

Here $$a = \tfrac{1}{2},\ r = \tfrac{2}{3},\ n = 4.$$
$$\therefore s = \frac{\frac{1}{2}\{1 - (\frac{2}{3})^4\}}{\frac{1}{3}} = 1\tfrac{11}{54}.$$

14. If a blacksmith uses seven nails in putting a shoe on a horse's foot, and receives 1 cent for the first nail, 2 cents for the second nail, and so on, what does he receive for putting on the shoe?

Here $$a = 1,\ r = 2,\ n = 7.$$
$$\therefore s = \frac{1\{2^7 - 1\}}{1} = 127.$$

Therefore he receives $1.27.

15. If a boy receives 2 cents for his first day's work, 4 cents for his second day, 8 cents for the third day, and so on for 12 days, what will his wages amount to?

Here $$a = 2,\ r = 2,\ n = 12.$$
$$\therefore s = \frac{2\,(2^{12} - 1)}{1} = 8190.$$

Therefore he receives $81.90.

16. If the population of a city is 10,000, and increases 10% a year for four years, what will be its population at the end of the four years? (Here $l = ar^4$.)

Here $$a = 10{,}000,\ r = \tfrac{11}{10},\ \text{and } l = ar^4.$$
$$\therefore l = 10{,}000 \times (\tfrac{11}{10})^4 = 14{,}641.$$

Exercise 78. Page 153.

Find the square root of:

1. $a^2 + 2ab + 2ac + b^2 + 2bc + c^2$.

$$\begin{array}{r|l}
 & a^2 + 2ab + 2ac + b^2 + 2bc + c^2 \;\underline{|\, a + b + c} \\
 & a^2 \\ \hline
2a + b & 2ab + 2ac + b^2 + 2bc + c^2 \\
 & 2ab \qquad\quad + b^2 \\ \hline
2a + 2b + c & 2ac \qquad\quad + 2bc + c^2 \\
 & 2ac \qquad\quad + 2bc + c^2 \\ \hline
\end{array}$$

2. $x^4 + 2x^3 + 3x^2 + 2x + 1.$

$$x^4 + 2x^3 + 3x^3 + 2x + 1 \,\underline{|\, x^2 + x + 1}$$
$$x^4$$
$$2x^2 + x \;|\; +2x^3 + 3x^2 + 2x$$
$$2x^3 + x^2$$
$$2x^2 + 2x + 1 \;|\; 2x^2 + 2x + 1$$
$$2x^2 + 2x + 1$$

3. $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4.$

$$x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4 \,\underline{|\, x^2 - 2xy + y^2}$$
$$x^4$$
$$2x^2 - 2xy \;|\; -4x^3y + 6x^2y^2$$
$$-4x^3y + 4x^2y^2$$
$$2x^2 - 4xy + y^2 \;|\; 2x^2y^2 - 4xy^3 + y^4$$
$$2x^2y^2 - 4xy^3 + y^4$$

4. $4a^4 - 12a^3b + 29a^2b^2 - 30ab^3 + 25b^4.$

$$4a^4 - 12a^3b + 29a^2b^2 - 30ab^3 + 25b^4 \,\underline{|\, 2a^2 - 3ab + 5b^2}$$
$$4a^4$$
$$4a^2 - 3ab \;|\; -12a^3b + 29a^2b^2$$
$$-12a^3b + 9a^2b^2$$
$$4a^2 - 6ab + 5b^2 \;|\; 20a^2b^2 - 30ab^3 + 25b^4$$
$$20a^2b^2 - 30ab^3 + 25b^4$$

5. $16x^6 + 24x^5y + 9x^4y^2 - 16x^3y^3 - 12x^2y^4 + 4y^6.$

$$16x^6 + 24x^5y + 9x^4y^2 - 16x^3y^3 - 12x^2y^4 + 4y^6 \,\underline{|\, 4x^3 + 3x^2y - 2y^3}$$
$$16x^6$$
$$8x^3 + 3x^2y \;|\; 24x^5y + 9x^4y^2$$
$$24x^5y + 9x^4y^2$$
$$8x^3 + 6x^2y - 2y^3 \;|\; -16x^3y^3 - 12x^2y^4 + 4y^6$$
$$-16x^3y^3 - 12x^2y^4 + 4y^6$$

6. $4x^6 - 4x^4y^2 + 12x^3y^3 + x^2y^4 - 6xy^5 + 9y^6.$

$$4x^6 - 4x^4y^2 + 12x^3y^3 + x^2y^4 - 6xy^5 + 9y^6 \,\underline{|\, 2x^3 - xy^2 + 3y^3}$$
$$4x^6$$
$$4x^3 - xy^2 \;|\; -4x^4y^2 + 12x^3y^3 + x^2y^4$$
$$-4x^4y^2 + x^2y^4$$
$$4x^3 - 2xy^2 + 3y^3 \;|\; 12x^3y^3 - 6xy^5 + 9y^6$$
$$12x^3y^3 - 6xy^5 + 9y^6$$

Exercise 79. Page 156.

Find the square root of :

1. 324.

3 24 (18
1
28) 224
224

2. 441.

4 41 (21
4
41) 41
41

3. 529.

5 29 (23
4
43) 129
129

4. 961.

9 61 (31
9
61) 61
61

5. 10.24.

10.24 (3.2
9
62) 124
124

6. 53.29.

53.29 (7.3
49
143) 429
429

7. 53,824.

5 38 24 (232
4
43) 138
129
462) 924
924

8. 616,225.

61 62 25 (785
49
148) 1262
1184
1565) 7825
7825

9. 1,500,625.

1 50 06 25 (1225
1
22) 50
44
242) 606
484
2445) 12225
12225

10. 346,921.

34 69 21 (589
25
108) 969
864
1169) 10521
10521

11. 31,371,201.

31 37 12 01 (5601
25
106) 637
636
11201) 11201
11201

12. 1,522,756.

1 52 27 56 (1234
1
22) 52
44
243) 827
729
2464) 9856
9856

Find to four decimal places the square root of :

13. 2.

2.00 00 00 00 (1.4142.....
1
24) 100
96
281) 400
281
2824) 11900
11296
28282) 60400
56564

14. 3.

3.00 00 00 00 (1.7320.....
1
27) 200
189
343) 1100
1029
3462) 7100
6924
34640) 17600

15. 5.

5.00 00 00 00 (2.2360....
4
42) 100
84
443) 1600
1329
4466) 27100
26796
44720) 30400

16. 6.

6.00 00 00 00 (2.4494.....
4
44) 200
176
484) 2400
1936
4889) 46400
44001
48984) 239900
195936

17. 0.5.

0.50 00 00 00 (0.7071.....
49
1407) 10000
9849
14141) 15100
14141

18. 0.9.

0.90 00 00 00 (0.9486.....
81
184) 900
736
1888) 16400
15104
18966) 129600
113796

19. $\frac{2}{3}$.

$\frac{2}{3} = 0.66.....$

```
     0.66 66 66 66 (0.8164.....
       64
  161) 266
       161
 1626) 10566
        9756
16324) 81066
       65296
```

20. $\frac{3}{4}$.

$\frac{3}{4} = 0.75.$

```
      0.75 00 00 00 (0.8660.....
        64
   166) 1100
         996
  1726) 10400
        10356
 17320) 4400
```

21. $\frac{4}{5}$.

$\frac{4}{5} = 0.8.$

```
      0.80 00 00 00 (0.8944.....
        64
   169) 1600
        1521
  1784) 7900
        7136
 17884) 76400
        71536
```

22. $\frac{5}{8}$.

$\frac{5}{8} = 0.625.$

```
      0.62 50 00 00 (0.7905.....
        49
   149) 1350
        1341
 15805) 90000
        79025
```

Exercise 80. Page 159.

Find the cube root of:

1. $x^3 + 3x^2y + 3xy^2 + y^3$.

$$\begin{array}{r|l} & x^3 + 3x^2y + 3xy^2 + y^3 \;\underline{|\, x + y} \\ & x^3 \\ 3x^2 + 3xy + y^2 & 3x^2y + 3xy^2 + y^3 \\ & 3x^2y + 3xy^2 + y^3 \end{array}$$

2. $8x^3 - 12x^2 + 6x - 1$.

$$\begin{array}{r|l} & 8x^3 - 12x^2 + 6x - 1 \;\underline{|\, 2x - 1} \\ & 8x^3 \\ 12x^2 - 6x + 1 & -12x^2 + 6x - 1 \\ & -12x^2 + 6x - 1 \end{array}$$

3. $8x^3 - 36x^2y + 54xy^2 - 27y^3$.

$$\begin{array}{r|l} & 8x^3 - 36x^2y + 54xy^2 - 27y^3 \;\underline{|\, 2x - 3y} \\ & 8x^3 \\ 12x^2 - 18xy + 9y^2 & -36x^2y + 54xy^2 - 27y^3 \\ & -36x^2y + 54xy^2 - 27y^3 \end{array}$$

4. $64a^3 - 144a^2x + 108ax^2 - 27x^3$.

$$
\begin{array}{r|l}
 & 64a^3 - 144a^2x + 108ax^2 - 27x^3 \;\underline{|\,4a - 3x} \\
 & 64a^3 \\
48a^2 - 36ax + 9x^2 & -144a^2x + 108ax^2 - 27x^3 \\
 & \underline{-144a^2x + 108ax^2 - 27x^3}
\end{array}
$$

5. $1 + 3x + 6x^2 + 7x^3 + 6x^4 + 3x^5 + x^6$.

$$
\begin{array}{l|l}
 & 1 + 3x + 6x^2 + 7x^3 + 6x^4 + 3x^5 + x^6 \;\underline{|\,1+x+x^2} \\
 & 1 \\
3 + 3x + x^2 & 3x + 6x^2 + 7x^3 \\
 & \underline{3x + 3x^2 + x^3} \\
3(1+x)^2 = 3+6x+3x^2 & 3x^2 + 6x^3 + 6x^4 + 3x^5 + x^6 \\
3(1+x)x^2 = \qquad 3x^2+3x^3 & \\
\underline{(x^2)^2 = \qquad\qquad\quad +x^4} & \\
3+6x+6x^2+3x^3+x^4 & \underline{3x^2 + 6x^3 + 6x^4 + 3x^5 + x^6}
\end{array}
$$

6. $x^6 - 3x^5 + 6x^4 - 7x^3 + 6x^2 - 3x + 1$.

$$
\begin{array}{l|l}
 & x^6 - 3x^5 + 6x^4 - 7x^3 + 6x^2 - 3x + 1 \;\underline{|\,x^2-x+1} \\
 & x^6 \\
3x^4 - 3x^3 + x^2 & -3x^5 + 6x^4 - 7x^3 + 6x^2 \\
 & \underline{-3x^5 + 3x^4 - x^3} \\
3(x^2-x)^2 = 3x^4-6x^3+3x^2 & 3x^4 - 6x^3 + 6x^2 - 3x + 1 \\
3(x^2-x) = \qquad 3x^2-3x & \\
\underline{(1)^2 = \qquad\qquad\quad +1} & \\
3x^4-6x^3+6x^2-3x+1 & \underline{3x^4 - 6x^3 + 6x^2 - 3x + 1}
\end{array}
$$

Exercise 81. Page 163.

Find the cube root of:

1. 46,656.

$$
\begin{array}{r|l}
 & 46\,656 \;\underline{|\,36} \\
 & 27 \\
3 \times (30)^2 = 2700 & 19656 \\
3\,(30 \times 6) = 540 & \\
6^2 = \underline{36} & \\
3276 & \underline{19656}
\end{array}
$$

2. 42,875.

$$
\begin{array}{r|l}
 & 42\,875 \;\underline{|\,35} \\
 & 27 \\
3 \times (30)^2 = 2700 & 15875 \\
3\,(30 \times 5) = 450 & \\
5^2 = \underline{25} & \\
3175 & \underline{15875}
\end{array}
$$

3. 91,125.

$$\begin{array}{rr|l}
 & 91\ 125 & \underline{45} \\
 & 64 & \\ \hline
3 \times (40)^2 = & 4800 & 27125 \\
3\,(40 \times 5) = & 600 & \\
5^2 = & \underline{25} & \\
 & 5425 & \underline{27125}
\end{array}$$

4. 274,625.

$$\begin{array}{rr|l}
 & 274\ 625 & \underline{65} \\
 & 216 & \\ \hline
3 \times (60)^2 = & 10800 & 58625 \\
3\,(60 \times 5) = & 900 & \\
5^2 = & \underline{25} & \\
 & 11725 & \underline{58625}
\end{array}$$

5. 110,592.

$$\begin{array}{rr|l}
 & 110\ 592 & \underline{48} \\
 & 64 & \\ \hline
3 \times (40)^2 = & 4800 & 46592 \\
3\,(40 \times 8) = & 960 & \\
8^2 = & \underline{64} & \\
 & 5824 & \underline{46592}
\end{array}$$

6. 258,474,853.

$$\begin{array}{rr|l}
 & 258\ 474\ 853 & \underline{637} \\
 & 216 & \\ \hline
3 \times (60)^2 = & 10800 & 42474 \\
3\,(60 \times 3) = & 540 & \\
3^2 = & \underline{9} & \\
 & 11349 & \underline{34047} \\
 & \underline{549} & 8427853 \\
3 \times (630)^2 = & 1190700 & \\
3\,(630 \times 7) = & 13230 & \\
7^2 = & \underline{49} & \\
 & 1203979 & \underline{8427853}
\end{array}$$

7. 109,215,352.

$$\begin{array}{rr|l}
 & 109\ 215\ 352 & \underline{478} \\
 & 64 & \\ \hline
3 \times (40)^2 = & 4800 & 45215 \\
3\,(40 \times 7) = & 840 & \\
7^2 = & \underline{49} & \\
 & 5689 & \underline{39823} \\
 & \underline{889} & 5392352 \\
3 \times (470)^2 = & 662700 & \\
3\,(470 \times 8) = & 11280 & \\
8^2 = & \underline{64} & \\
 & 674044 & \underline{5392352}
\end{array}$$

8. 259,694,072.

$$
\begin{array}{rr|l}
 & & 259\ 694\ 072\ \underline{|638} \\
 & & 216 \\
3 \times (60)^2 = & 10800 & 43694 \\
3\,(60 \times 3) = & 540 & \\
3^2 = & \underline{9} & \\
 & 11349 & \underline{34047} \\
 & 549 & 9647072 \\
3 \times (630)^2 = & 1190700 & \\
3\,(630 \times 8) = & 15120 & \\
8^2 = & \underline{64} & \\
 & 1205884 & \underline{9647072}
\end{array}
$$

9. 127,263,527.

$$
\begin{array}{rr|l}
 & & 127\ 263\ 527\ \underline{|503} \\
 & & 125 \\
3 \times (500)^2 = & 750000 & 2263527 \\
3\,(500 \times 3) = & 4500 & \\
3^2 = & \underline{9} & \\
 & 754509 & \underline{2263527}
\end{array}
$$

10. 385,828,352.

$$
\begin{array}{rr|l}
 & & 385\ 828\ 352\ \underline{|728} \\
 & & 343 \\
3 \times (70)^2 = & 14700 & 42828 \\
3\,(70 \times 2) = & 420 & \\
2^2 = & \underline{4} & \\
 & 15124 & \underline{30248} \\
 & 424 & 12580352 \\
 & 1555200 & \\
3\,(720 \times 8) = & 17280 & \\
8^2 = & \underline{64} & \\
 & 1572544 & \underline{12580352}
\end{array}
$$

11. 1879.080904.

$$
\begin{array}{rr|l}
 & & 1\ 879.080\ 904 \ \underline{|\,12.34} \\
 & & 1 \\
\hline
3 \times (10)^2 = & 300 & 879 \\
3\,(10 \times 2) = & 60 & \\
2^2 = & \underline{4} & \\
 & 364 & \underline{728} \\
 & 64 & 151080 \\
\hline
3 \times (120)^2 = & 43200 & \\
3\,(120 \times 3) = & 1080 & \\
2^2 = & \underline{9} & \\
 & 44289 & \underline{132867} \\
 & 1089 & 18213904 \\
\hline
3 \times (1230)^2 = & 4538700 & \\
3\,(1230 \times 4) = & 14760 & \\
4^2 = & \underline{16} & \\
 & 4553476 & \underline{18213904}
\end{array}
$$

12. 1838.265625.

$$
\begin{array}{rr|l}
 & & 1\ 838.265\ 625 \ \underline{|\,12.25} \\
 & & 1 \\
\hline
3 \times (10)^2 = & 300 & 838 \\
3\,(10 \times 2)^2 = & 60 & \\
2^2 = & \underline{4} & \\
 & 364 & \underline{728} \\
 & 64 & 110265 \\
\hline
3 \times (120)^2 = & 43200 & \\
3\,(120 \times 2) = & 720 & \\
2^2 = & \underline{4} & \\
 & 43924 & \underline{87848} \\
 & 724 & 22417625 \\
\hline
3 \times (1220)^2 = & 4465200 & \\
3\,(1220 \times 5) = & 18300 & \\
5^2 = & \underline{25} & \\
 & 4483525 & \underline{22417625}
\end{array}
$$

Find to four decimal places the cube root of:

13. 0.01.

	0.010 000 000 000	0.2154.....
	8	
$2 \times (20)^2 = 1200$	2000	
$3\,(20 \times 1) = 60$		
$1^2 = 1$		
1261	1261	
61	739000	
$3 \times (210)^2 = 132300$		
$3\,(210 \times 5) = 3150$		
$5^2 = 25$		
135475	677375	
3175	61625000	
$3 \times (2150)^2 = 13867500$		
$3\,(2150 \times 4) = 25800$		
$4^2 = 16$		
13893316	55573264	

14. 0.05.

	0.050 000 000 000	0.3684.....
	27	
$3 \times (30)^2 = 2700$	23000	
$3\,(30 \times 6) = 540$		
$6^2 = 36$		
3276	19656	
576	3344000	
$3 \times (360)^2 = 388800$		
$3\,(360 \times 8) = 8640$		
$8^2 = 64$		
397504	3180032	
8704	163968000	
$3 \times (3680)^2 = 40627200$		
$3\,(3680 \times 4) = 44160$		
$4^2 = 16$		
40671376	162685504	

15. 0.2.

		$0.200\ 000\ 000\ 000 \,\lfloor\, 0.5848\ldots\ldots$
		125
$3 \times (50)^2 =$	7500	75000
$3\,(50 \times 8) =$	1200	
$8^2 =$	64	
	8764	70112
	1264	4888000
$3 \times (580)^2 =$	1009200	
$3\,(580 \times 4) =$	6960	
$4^2 =$	16	
	1016176	4064704
	6976	823296000
$3 \times (5840)^2 =$	102316800	
$3\,(5840 \times 8) =$	140160	
$8^2 =$	64	
	102457024	819656192

16. 4.

		$4.000\ 000\ 000\ 000 \,\lfloor\, 1.5874\ldots\ldots$
		1
$3 \times (10)^2 =$	300	3000
$3\,(10 \times 5) =$	150	
$5^2 =$	25	
	475	2375
	175	625000
$3 \times (150)^2 =$	67500	
$3\,(150 \times 8) =$	3600	
$8^2 =$	64	
	71164	569312
	3664	55688000
$3 \times (1580)^2 =$	7489200	
$3\,(1580 \times 7) =$	33180	
$7^2 =$	49	
	7522429	52657003
	33229	3030997000
$3 \times (15870)^2 =$	755570700	
$3\,(15870 \times 4) =$	190440	
$4^2 =$	16	
	755761156	3023044624

17. 10.

	10.000 000 000 000 | 2.1544.....
	8
$3 \times (20)^2 = 1200$	2000
$3\,(20 \times 1) = 60$	
$1^2 = 1$	
1261	1261
61	739000
$3 \times (210)^2 = 132300$	
$3 \times (210 \times 5) = 3150$	
$5^2 = 25$	
135475	677375
3175	61625000
$3 \times (2150)^2 = 13867500$	
$3\,(2150 \times 4) = 25800$	
$4^2 = 16$	
13893316	55573264
25816	6051736000
$3 \times (21540)^2 = 1391914800$	
$3 \times (21540 \times 4) = 258480$	
$4^2 = 16$	
1392173296	5568693184

18. 87.

	87.000 000 000 000 | 4.4310.....
	64
$3 \times (40)^2 = 4800$	23000
$3\,(40 \times 4) = 480$	
$4^2 = 16$	
5296	
496	21184
$3 \times (440)^2 = 580800$	1816000
$3(440 \times 3) = 3960$	
$3^2 = 9$	
584769	
3969	1754307
$3 \times (4430)^2 = 58874700$	61693000
$3\,(4430 \times 1) = 13290$	
$1^2 = 1$	
58887991	
13291	58887991
$3 \times (44310)^2 = 5890128300$	2805009000

19. 2.5.

		$2.500\ 000\ 000\ 000 \mid 1.3572\ldots\ldots$
		1
$3 \times (10)^2 =$	300	1500
$3\,(10 \times 3) =$	90	
$3^2 =$	9	
	399	1197
	99	303000
$3 \times (130)^2 =$	50700	
$3\,(130 \times 5) =$	1950	
$5^2 =$	25	
	52675	263375
	1975	39625000
$3 \times (1350)^2 =$	5467500	
$3\,(1350 \times 7) =$	28350	
$7^2 =$	49	
	5495899	38471293
	28399	1153707000
$3 \times (13570)^2 =$	552434700	
$3\,(13570 \times 2) =$	81420	
$2^2 =$	4	
	552516124	1105032248

20. 2.05.

		$2.050\ 000\ 000\ 000 \mid 1.2703\ldots\ldots$
		1
$3 \times (10)^2 =$	300	1050
$3\,(10 \times 2) =$	60	
$2^2 =$	4	
	364	728
	64	322000
$3 \times (120)^2 =$	43200	
$3\,(120 \times 7) =$	2520	
$7^2 =$	49	
	45769	320383
	2569	1617000000
$3 \times (12700)^2 =$	483870000	
$3\,(12700 \times 3) =$	114300	
$3^2 =$	9	
	483984309	1451952927

21. 3.02.

	3.020 000 000 000 | 1.4454...
	1
$3 \times (10)^2 = 300$	2020
$3\,(10 \times 4) = 120$	
$4^2 = 16$	
436	1744
136	276000
$3 \times (140)^2 = 58800$	
$3\,(140 \times 4) = 1680$	
$4^2 = 16$	
60496	241984
1696	34016000
$3 \times (1440)^2 = 6220800$	
$3\,(1440 \times 5) = 21600$	
$5^2 = 25$	
6242425	31212125
21625	2803875000
$3 \times (14450)^2 = 626407500$	
$3\,(14450 \times 4) = 173400$	
$4^2 = 16$	
626580916	2506323664

22. $\frac{2}{3}$.

$\frac{2}{3} = 0.666.....$

	0.666 666 666 666 | 0.8735.....
	512
$3 \times (80)^2 = 19200$	154666
$3\,(80 \times 7) = 1680$	
$7^2 = 49$	
20929	146503
1729	8163666
$3 \times (870)^2 = 2270700$	
$3\,(870 \times 3) = 7830$	
$3^2 = 9$	
2278539	6835617
7839	1328049666
$3 \times (8730)^2 = 228638700$	
$3\,(8730 \times 5) = 130950$	
$5^2 = 25$	
228769675	1143848375

23. $\frac{3}{4}$.

$\frac{3}{4} = 0.75$

$$
\begin{array}{rr|l}
 & & 0.750\ 000\ 000 \,\underline{|\, 0.9085.....} \\
 & & \underline{729} \\
3 \times (900)^2 = & 2430000 & 21000000 \\
3\,(900 \times 8) = & 21600 & \\
8^2 = & \underline{64} & \\
 & 2451664 & \underline{19613312} \\
 & \underline{21664} & 1386688000 \\
3 \times (9080)^2 = & 247339200 & \\
3\,(9080 \times 5) = & 136200 & \\
5^2 = & \underline{25} & \\
 & 247475425 & \underline{1237377125}
\end{array}
$$

24. $\frac{9}{11}$.

$\frac{9}{11} = 0.8181818181 81.$

$$
\begin{array}{rr|l}
 & & 0.818\ 181\ 818\ 181 \,\underline{|\, 0.9352.....} \\
 & & \underline{729} \\
3 \times (90)^2 = & 24300 & 89181 \\
8\,(90 \times 3) = & 810 & \\
3^2 = & \underline{9} & \\
 & 25119 & \underline{75357} \\
 & \underline{819} & 13824818 \\
3 \times (930)^2 = & 2594700 & \\
3\,(930 \times 5) = & 13950 & \\
5^2 = & \underline{25} & \\
 & 2608675 & \underline{13043375} \\
 & \underline{13975} & 781443181 \\
3 \times (9350)^2 = & 262267500 & \\
3\,(9350 \times 2) = & 56100 & \\
2^2 = & \underline{4} & \\
 & 262323604 & \underline{524647208}
\end{array}
$$

ADVERTISEMENTS

MATHEMATICS.

WENTWORTH'S SERIES.

TWO words indicate the two leading characteristics of these remarkable books: scholarship and text-book availability. They combine mathematical brilliancy with all the elements of school-room popularity. The author's aim was to be helpful. He has made a study of his subjects with the psychology and probable capacity of the students constantly in mind. His books enable the average student not merely to "learn something," but to MASTER the study, and at the same time they give the brighter ones plenty to exercise their faculties. There is no attempt to make a parade of learning. On the other hand, there is no absolution from solid work. Everything is made simple, practical, direct, and thorough. Mental energy is economized. The teacher has to use his strength only in the necessary work of teaching, and the pupil his only in the necessary work of acquiring.

To learn by doing, and to learn one step thoroughly before the next is attempted, are the chief elements of the method, and yet no books give pupil or teacher less of the tread-mill feeling. The consciousness of mastery constantly cheers and invigorates the student, while the teacher has the satisfaction of wielding an instrument fitted to the hand. All will recognize in these characteristics the marks of ideal text-books. The testimony of use may be summed up in one word, *Satisfaction.*

The following figures speak volumes for the success of the Wentworth books:

In January, 1894, according to the most accurate information obtainable, Wentworth's Series, in whole or in part, was in use by over 220,000 pupils in the secondary schools alone of the United States. In the three representative states, Massachusetts, New York, and Illinois, 41,306 pupils were using Wentworth, while only 9,191 were using the next most popular book. For New York State, there are official figures — those of the Regents of the University. These show that for 1892–93, of the schools under the control of the Regents, the number using Wentworth was: Algebra, 178; Plane Geometry, 280; Solid Geometry, 112; and the numbers using the next most popular book were respectively 65, 21, and 9.

Introductory Books.

Wentworth's Primary Arithmetic. ***Wentworth's Elementary Arithmetic.***
Introduction price, each 30 cents.

Wentworth's Grammar School Arithmetic.
Introduction price, 65 cents.

This, with either the Primary or the Elementary Arithmetic, forms a complete course for Common Schools, and is believed to present the best known methods in the most attractive, available, and practical form.

Wentworth and Reed's First Steps in Number.
For prices, see List at the beginning of this Catalogue; for full description, see Common School Catalogue.

A High School Arithmetic.

(Wentworth and Hill's Practical Arithmetic.)

By G. A. WENTWORTH, Professor of Mathematics, Phillips Exeter Academy, and the late Dr. THOMAS HILL, Portland, Me. 12mo. Roan back. 367 pages. Mailing price, $1.10; Introduction, $1.00. Answers free, on teacher's order.
Same. Abridged Edition. For Grammar Schools. 288 pages (*including Answers*). Mailing price, $1.00; for introduction, 75 cents.

THIS book is intended for high and normal schools and academies. It assumes that the pupil has some knowledge of the simple processes of Arithmetic, and aims to develop his power over practical questions, as well as to increase his facility in computing. Such problems are selected as are calculated to interest the pupil, and lead him to independent thought and discovery. The problems cover a wide range of subjects, and are particularly adapted to general mental discipline, to preparation for higher studies, mechanical work, business, or professional life.

J. M. Peirce, *Prof. of Mathematics, Harvard College:* It is clear, straightforward, sound, careful, and abundantly supplied with well-chosen examples. I would especially commend the pains that have been taken to set forth intelligibly the *shortest and easiest methods of work*, such as are actually used by the best computers, and the inclusion of various principles and rules, often neglected by mere text-book compilers, which go far to form a really scientific habit of thought in the young student.

H. Carr Pritchett, *Prof. of Mathematics, Sam Houston Normal Institute, Tex.:* Wentworth and Hill's Arithmetic is exceedingly practical. All useless matter has been discarded, and the fundamental principles so clearly set forth that it is a model of *simplicity* and *brevity*.

Wentworth's School Algebra.

By G. A. Wentworth, Professor of Mathematics in Phillips Exeter Academy. Half morocco. v + 362 pages. Mailing price, $1.25; for introduction, $1.12. *Answers in pamphlet form, free, on teachers' orders.*

IN this book the author has availed himself of his own experience in writing and teaching the elements of Algebra, and of the experience of hundreds of others. Where improvement was possible, it has been made.

The School Algebra is offered as exactly right for the usual high school and academic courses. It gives a thorough and practical treatment of the principles of Algebra up to and including the binomial theorem, and is strictly in line throughout with the author's College Algebra. For college preparation it is particularly well suited.

The problems in this book are nearly all new, either original or selected from recent examination papers, and are graded with the utmost care. They are sufficient in number to illustrate and fix all the principles, and interesting and varied enough to hold the student's attention through the book. The passage from Arithmetic to Algebra is made easy, and the advantages of using letters clearly pointed out. The treatment of fractions has been further simplified. Radicals precede quadratics. There is at the end a carefully made collection of miscellaneous examples, covering nearly every principle of Algebra. This, with the author's College Algebra, makes a complete and consistent course.

F. D. Sherman, *Principal High School, Bay City, Mich.:* We think it an excellent text-book, or we should not at present be using it.

Theodore L. Sewall, *Girls' Class. School, Indianapolis:* An admirable book, like all of the Wentworth Series.

George Gilbert, *Chester Academy, Pa.:* The best book for its grade yet issued by an American publisher.

Richard H. Lewis, *Judson College, N.C.:* It is fresh, strong, really invigorating, and requires thought to master.

George E. Gay, *High School, Malden, Mass.:* Better adapted to use in high schools than any other.

Jno. T. Buchanan, *High School, Kansas City, Mo.:* We find it to be a first-class book, a book just suited to our work.

T. W. Palmer, *Prof. of Mathematics, University of Alabama:* An admirable work.

C. D. Schmitt, *University of Tennessee:* For the work intended, I do not think it can be surpassed.

L. R. Hunt, *High School, Troy, N.Y.:* It meets my desires completely.

Wentworth's College Algebra.

By G. A. WENTWORTH, Professor of Mathematics in Phillips Exeter Academy. Half morocco. 500 pages. Mailing price, $1.65; for introduction, $1.50. *Answers in pamphlet form, free, on teachers' orders.*

THIS is what its name indicates, a text-book for colleges and scientific schools. The first part is simply a concise review of the principles of Algebra preceding quadratics, with enough examples to illustrate and enforce the principles. Room is thus left for a full discussion of the higher topics. The endeavor has been to give in matter and methods the best training in algebraic analysis at present attainable. The work covers a full year, but by omitting starred sections and problems, the instructor can arrange a half-year course. Many professors helped with suggestions to make the College Algebra fit prevailing requirements.

William Beebe, *Assistant Prof. of Mathematics and Astronomy, Yale College:* I find it characterized by the clearness and method of all Professor Wentworth's books, and am particularly struck with the amount of matter in the Algebra.

Dascom Greene, *Prof. of Mathematics and Astronomy, Rensselaer Polytechnic Institute, Troy, N.Y.:* The methods of treatment seem to be generally judicious, and the style attractive, and to the student well grounded in the elements it furnishes an excellent practical course in the higher branches of the subject.

E. S. Crawley, *Professor of Mathematics, University of Pennsylvania:* It is a great improvement over —— for college use, and in its way seems to me to leave nothing to be desired.

T. C. Leonard, S. J., *Prof. of Mathematics, St. Ignatius College, San Francisco:* The book, in my opinion, is a model Algebra, distinguished for its clearness of explanation and the eminently practical nature of its matter.

J. C. Glashan, *Inspector of Public Schools, Ottawa, Canada:* I am satisfied I can unqualifiedly recommend it.

Henry Ray Warner, *Prof. of Mathematics, Mt. Union College, O.:* Both as to subject-matter and style, I regard it as the most perfect Algebra I have ever examined.

Henry C. King, *Prof. of Mathematics, Oberlin College, O.:* The chapter on Determinants I think a specially valuable feature, which alone would tell strongly in favor of the book for college use.

E. P. Thompson, *Prof. of Mathematics, Geneva College, Pa.:* This is such a work as the college student ought to use.

Wentworth's Higher Algebra.

By G. A. Wentworth, Professor of Mathematics in Phillips Exeter Academy. Half morocco. 528 pages. Mailing price, $1.55; for introduction, $1.40. *Answers in pamphlet form, free, on teachers' orders.*

THIS work is designed to prepare thoroughly for colleges and scientific schools, and to furnish in addition what is needed for the *general student* in such institutions.

Preparatory schools and academies of high grade, especially such as give the pupils a thorough preliminary drill in arithmetic, normal schools, seminaries, and many colleges, will find it specially suited to their requirements.

It is substantially equivalent to the author's Complete Algebra, but is believed to be in many respects better. The arrangement and the treatment of the topics have been revised, and fuller explanations have been given. It is more complete than the School Algebra, inasmuch as it takes up the topics usually included in higher Algebra. It gives a fuller treatment of the simpler elements of Algebra than the College Algebra, and not so elaborate a treatment of the more advanced portions.

In a word, the Higher Algebra provides *in a single book a complete course parallel* to the course provided by the School and College Algebras together.

Davis Garber, *Professor of Mathematics and Astronomy, Muhlenberg College, Allentown, Pa.:* It seems to meet the wants of a general college course better than any of the author's other works on the same subject. With its elementary and advanced parts, its excellent arrangement and clear treatment of topics, it is certainly a very desirable text-book.

Geo. V. Buchanan, *Prof. of Mathematics, Southern Illinois State Normal University, Carbondale, Ills.:* I have used matter from it for special drills and tests in my classes, and I am of the opinion that it is the best book for advanced work I have ever seen.

L. F. Bickford, *Daniel Baker College, Brownwood, Texas:* One could scarcely see room for improvement upon Wentworth's Algebras as previously published, but the author in his Higher Algebra has solved the problem of improvement with remarkable success. In method and matter, and logical arrangement, and in vigorous analysis, it is well nigh perfect, and it easily leads as a text-book.

Milton L. Comstock, *Prof. of Mathematics, Knox College, Galesburg, Ills.:* The explanations are short and to the point always; the examples are numerous and well selected, and the general scope of the treatise is ample for the demand of a college course.

Wentworth's Elements of Algebra.

By GEORGE A. WENTWORTH, Professor of Mathematics, Phillips Exeter Academy, and author of *Geometry*, *Trigonometry*, etc. Half morocco. x + 325 pages. Mailing price, $1.25; for introduction, $1.12. *Answers bound separately in pamphlet form.*

THIS book is designed for high schools and academies, and contains an ample amount for admission to any college. In grade and amount it is exactly equivalent to the *School Algebra*, and as the problems are entirely different, it may be used as an alternative when a change of problems is found desirable.

Wentworth's Complete Algebra.

Includes the subjects usually taught in Colleges. Half morocco. 525 pages. Mailing price, $1.55; for introduction, $1.40. *Answers bound separately in pamphlet form.*

THIS work consists of the author's Elementary Algebra, with about one hundred and eighty-five pages additional, and it may be used as an alternative for the *Higher Algebra* to obtain a new set of problems.

Wentworth's First Steps in Algebra.

Half morocco. Specially prepared for the use of grammar schools.

Wentworth's Shorter Course in Algebra.

Half morocco. 258 pages. By mail, $1.10; for introduction, $1.00. *Answers in pamphlet form, free, on teachers' orders.*

Algebraic Analysis.

By G. A. WENTWORTH, A.M., Professor of Mathematics in Phillips Exeter Academy; J. A. MCLELLAN, LL.D., Inspector of Normal Schools, and Conductor of Teachers' Institutes for Ontario, Canada; and J. C. GLASHAN, Inspector of Public Schools, Ottawa, Canada. Part I. concluding with Determinants. 12mo. Half leather. x + 418 pages. By mail, $1.60; to teachers and for introduction, $1.50.

THIS work is intended to supply students of Mathematics with a well-filled storehouse of solved examples and unsolved exercises in the application of the fundamental theorems and processes of pure Algebra.

Wentworth's New Plane Geometry.

By George A. Wentworth, Teacher of Mathematics, Phillips Exeter Academy, N.H. 12mo. x + 242 pages. Mailing Price, 85 cents; Introduction, 75 cents.

Wentworth's New Plane and Solid Geometry.

By George A. Wentworth, Phillips Academy, Exeter, N.H. 12mo. Half morocco. xi + 386 pages. Mailing price, $1.40; Introduction, $1.25. The book now includes a treatise on Conic Sections (Book IX.).

ALL the distinguishing characteristics of the first edition have been retained. The subject is treated as a branch of practical logic, the object of which is to detect and state with precision the successive steps from premise to conclusion.

In each proposition a concise statement of what is given is printed in one kind of type, of what is required in another, and the demonstration in still another. The reason for each step is indicated in small type between that step and the one following; and the author thus avoids the necessity of interrupting the process of demonstration to cite a previous proposition. The number of the section on which the reason depends, is, however, placed at the side of the page; and the pupil should be prepared, when called upon, to give the proof of each reason. Each distinct assertion in the demonstrations and each particular direction in the construction of the figures begins a new line, and in no case is it necessary to turn the page in reading a demonstration.

In the new edition will be found a few changes in the order of the subject-matter. Some of the demonstrations have been given in a more concise and simple form. The diagrams, with which especial care was taken originally, have been re-engraved and materially improved. The shading, which has been added to many of the figures, has proved a great help to the constructive imagination of pupils. The theory of limits — the value of which the author emphasizes — has been presented in the simplest possible way, and its application made easy of comprehension.

But the great feature of this edition is the introduction of nearly seven hundred original exercises, consisting of theorems, problems of construction, and problems of computation, carefully graded and adapted to beginners in Geometry.

Wentworth & Hill's Exercises in Algebra.

I. Exercise Manual. 12mo. Boards. 232 pages. Mailing price, 40 cents; for introduction, 35 cents. II. Examination Manual. 12mo. Boards. 159 pages. Mailing price, 40 cents; for introduction, 35 cents. *Both in one volume*, 70 cents. *Answers to both together*, 25 cents.

THE first part (Exercise Manual) contains about 4500 problems classified and arranged according to the usual order of text-books in Algebra; and the second part (Examination Manual) contains nearly 300 progressive examination-papers, well adapted to cultivate skill and rapidity in solving problems.

British Mail: All engaged in the practical work of education will appreciate these Manuals, as they are calculated to save the master much precious time and labor, and to give his students the benefit of progressive and carefully thought-out exercises.

Wentworth & Hill's Exercises in Geometry.

12mo. Cloth. 255 pages. Mailing price, 80 cents; for introduction, 70 cents. *Answers are included in the volume.*

THE exercises consist of a great number of easy problems for beginners, and enough harder ones for more advanced pupils. The problems of each section are carefully graded, and some of the more difficult sections can be omitted without destroying the unity of the work.

Wentworth & Hill's Examination Manual in Geometry.

12mo. Cloth. iii + 138 pages. Mailing price, 55 cents; for introduction, 50 cents.

THE aim of the authors has been to give some elementary but much needed instruction in the art of handling original theorems and problems; and to supply a series of graded test-papers in Geometry which can be used not merely as *tests* of knowledge actually obtained, but also as a *means* of developing and strengthening the power to originate and carry on a logical train of thought; in other words, as a means of gaining the great object for which Geometry ought to be studied. The uses to which this book can be put must be apparent to every teacher of the subject.